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INFORMATION AGGREGATION BY MAJORITY RULE: THEORY AND EXPERIMENTS[1]

by

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Abstract

Although majority rule has limited value for aggregating conflicting preferences, it offers promise for aggregating decentralized information. The Condorcet Jury Theorem (CJT) states that majoritarian collective judgments can improve upon the accuracy of the judgements of the constituent individual voters. Recently, it has been argued that the CJT assumes implicitly that *each* vote reveals the voter's private information, and that such behavior *by all voters* is not usually a Nash equilibrium. Some voters may have reason to ignore their private information, and majority rule voting may fail to realize the judgmental synergies predicted. We also prove that there exists a Nash equilibrium at which the information aggregation synergies of majority rule surpass those predicted by the CJT. We report on experiments testing whether (a) the voting by individuals reflects their information, and (b) majority rule generates the synergy predicted by the CJT. The results indicate that the judgmental benefits of majority rule are robust. Groups do better than individuals even in situations in which the attractiveness of non-informative voting should be high.

1/ We especially wish to thank Andrew Herr and also David Austen-Smith and Jefferey S. Banks, Keith Dougherty, John Guyton, and Marek Kaminski for their comments. Earlier drafts of this paper were delivered at Shambaugh Conference, at the Department of Political Science of the University of Iowa, in May, 1995, and also at the Economic Science Assoc., Oct. 13, 1995. Tucson, Az.

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INFORMATION AGGREGATION BY MAJORITY RULE: THEORY AND EXPERIMENTS

> . . . many individual voters act in odd ways indeed; yet in the large the electorate behaves about as rationally and responsibly as we should expect, given the clarity of the alternatives presented to it and the character of the information available to it. (Key 1966, p. 7).

Most regulatory goals have a basis in legislative or other political. But legislative decisions are usually insufficient to flesh out regulatory policy. So it is that the substantive details of regulation are often identified by further decision making. In countries such as ours, these regulatory outcomes often reflect expert opinion. These opinions are rarely those of a single individual expert. Rather, the decisive intervention reflects the considered, aggregated judgements of a panel of experts looking at information available to them as a group, and understood in terms of the information they each hold privately (for example, the sort of information they have learned to process on the basis of their education and experience). In its simplest form, this is done via some form of voting, usually majority rule (MR).

Over the last decades, scholars have forcefully explored the inadequacies of majority rule (Arrow). When aggregating conflicting preferences, majority rule often results in intransitive or arbitrary choices (Condorcet, Plott). The implications include potential instability, manipulability, or even chaos (McKelvey, Schofield). For Riker, the lesson of this theoretical fact was obvious: democratic government is best when it undertakes the least. That is, democracy should constrain majority rule, rather than empower it. Even if this may have some credence with regard to governance as a whole, the proposal would be hardly adequate to the problem of for regulatory purposes.

The pessimistic results about MR intransitivity assume that citizens are voting to resolve preference-based conflicts. As an example: "Shall we take $1 from Ms. Smith and split it between the rest of the voters?" Questions such as these are about preferences, and therefore are not true or false. The social choice literature on which Riker based his case for limited government is based on majority rule's inability to deal with these preference-based conflicts in a stable fashion.[1]

Other propositions <u>can</u> be either true or false: "The defendant is guilty;" "Smoking cigarettes cause cancer;" "An increase in short-term interest rates increases the chance of a recession;" or, "The article submitted to this journal is correct and of sufficient importance to justify publication." If we knew the truth of such statements, then we would be more likely to have similar preferences about the appropriate actions.

1/ For a rigorous defense of majority rule in spite of this, see N. Miller (1983).

If the accused is innocent, few would object to her release from jail, and if an increase in interest rates will create a recession, then the Central Bank should not raise the interest rates. Such issues are ones that Coleman and Ferejohn term "epistemic."[2]

In regulatory decisions often information is diffused among experts,[3] it is important to make correct judgments about epistemic statements. To make effective policy in the presence of market failure, for example,[4] it is necessary to incorporate bits of information that are dispersed among society's experts and its citizenry.

In this paper we test how well majority rule aggregates disparate beliefs about epistemic propositions. This is needed to understand MR's effectiveness in the aid of developing good public policy in the real world. If we find that there are substantial inadequacies in aggregating epistemic judgements via majority rule it would have substantial consequences for how we ought to develop our policies.

The paper proceeds as follows. Of course, we begin by understanding the main theory's (i.e. the Condorcet Jury Theorem, CJT) linking of majoritarian rules for the aggregation of 'epistemic judgements' and 'good outcomes.' After sketching the theory, we discuss an experimental setting which both illustrates how the theory works, and which we will use to test the conjectures of the paper. We also review an alternative argument by Austin Smith and Banks (ASB). ASB shows that under specifiable conditions, the CJT is inconsistent with Nash equilibrium behavior. The relation between Nash and the CJT is explored. Experimental results are given and conclusions are drawn.

The Condorcet Jury Theorem (CJT)

The most germane, and hopeful, arguments linking majority rule with epistemic judgements come from Condorcet, who developed his arguments more than 200 years ago. Condorcet proved that MR voting can have advantages in discovering truth. The argument is simple: Assume a group of imperfectly informed voters, each with a probability $p > .5$ of being correct in a binary decision situation. If the votes are

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2/ Epistemic statements like statements in positive theory are either true or false. Preference-based statements being normative are neither.

3/ In a society policy-relevant information is broadly diffused among its citizens (Hayek).

4/ Much of the theoretical work on markets in recent years has emphasized the role that markets play in the aggregation of dispersed information, which is incorporated into the price of risky stocks (Varian). But markets sometimes fail due to monopoly, asymmetric information, or externalities.

statistically independent, then a majority of the group of voters would be correct more often than any one of its members. Furthermore, the probability that the group is correct increases and approaches 1 as the size of the group increases.

For example: assume that each of 3 voters gets enough information to be correct with probability .6 in deciding which side of an epistemic issue is correct. Then a majority of the voters will be correct with probability .648.[5] Majority rule creates judgmental synergy; as a truth-discerning entity, **the majority rule body is better than any of its members**. Furthermore, the probability that the majority is correct will approach 100% as the size of the group approaches infinity.

The original theorem makes some constraining assumptions. It is unrealistic to require either that all voters have the same probability of being correct, or that their votes be independent. Yet, when these assumptions are not met, the results of the theorem may not hold. For example, a majority of a panel of perfectly correlated voters will have the same chance of being correct as any one of the voters. Indeed, depending on individual levels of expertise and correlations among individuals, it is *possible* that majority rule can result in less effective judgments than those of any individual (Ladha, 1992).[6]

It is important to note that Condorcet Jury Theorem does not suggest that the simple majority-rule voting is the optimum rule. The optimum voting rule will depend on parameters (including individual levels of expertise and correlation among voters) which will change from issue to issue. For some parameters, a super-majority rule may be the optimum rule. Or, if one person (for example, a surgeon) is more informed than all the others combined, then it may be preferable to defer to the expert. But clearly, it is impossible to change the voting rule from issue to issue. As a constitutional question, society must decide on a mechanism

5/ This can be seen by calculating the probability of the majority being right. Some majority could be right either by all three voters voting correctly simultaneously, or by a majority of two voters voting correctly. With a probability of $.6^3$ (= .216) all 3 will vote correctly. The probability of only a specific pair of them voting correctly is (.6)*(.6)*(.4) = .144. There are three such pairs, so altogether some majority will be correct 3*(.144) + (.216) = .648. Similarly, a majority of nine voters will be correct with probability .733. This example assumes that the voters are independent.

6/ The apparent success of real democracies, however, suggests that the CJT should hold under assumptions far weaker than those Condorcet contemplated, and it does. The theorem has been generalized to admit heterogeneous pi (Grofman, Owen and Feld, 1983; Boland, 1989), and statistically correlated voting (Ladha, 1992, 1993, 1995; Berg 1993). Indeed, this literature offers insights into the virtues of MR voting under conditions of diversity (N. Miller, 1986; Ladha and G. Miller, 1995). In brief, the modern development of the CJT has, to this point, led to a reformulation which gives it a firmer foundation and increased applicability.

for making policy judgments on multiple issues, without knowing the parameters that would determine the optimal rule for making those judgments.

The democratic rule of most interest is majority rule. The question is how well groups make majority rule judgments as compared to the individuals composing that group. We address the question both theoretically and experimentally. Theoretically, we examine the sensitivity of the CJT result to Nash equilibrium behavior. Experimentally, we report the results of the first controlled laboratory experiments testing whether or not majority rule does result in an improvement over individual judgments.

The Experimental Setting

In analyzing the theoretical problem, it will be useful to refer to the experimental setting as an example. The experimental conditions are designed to provide maximal insights into theoretical concerns.

The experiment tests the behavior of experimental subjects. The subjects are to guess the color of a marble which is to be hidden from their view. Prior to the hiding of the marble, the subjects are shown two urns: one marked "60W" containing 60 white and 40 black marbles; the other marked "100B" containing 100 marbles, all black. Each group of three voters is informed that the hidden marble will be drawn from the "60W" urn and then hidden in an envelope. Thus, the subjects know that the hidden marble has a 60% chance of being white and a 40% chance of being black. The set of possible colors of the hidden marble (i.e. the possible states of the world) can be denoted {W, B}, only one of which is True. Therefore, each of the voters may be presumed to have a set of commonly held prior beliefs *(priors)*: {P(W), P(B)}. These beliefs are ex ante probabilities regarding the state of the world prior to the receipt of the signal. With only two states of the world, P(W) = 1 - P(B). In our experimental setting, P(W) = .6.

Each voter is to vote as they wish regarding the color of the hidden ball: either "white" or "black". That is, the voters each have a set of possible actions {*w,b*} one of which is to be taken by each voter. The members of the group will each earn an identical reward (from $1 to $15) each time that the group predicts the color of the hidden marble accurately.

If each voter voted on the basis of her prior beliefs only, then each would presumably vote white, and be correct 60% of the time. This being true for all voters, all would unanimously vote white, implying that they would be correct only 60% of the time--the same record as each of the individuals. Clearly, majority rule offers no advantage when the group is perfectly homogenous in its beliefs.

However, if each voter has private information, then an improvement is possible (but not necessary). In the experimental setting, all voters are told that they will receive a private signal which will give them a clue about the hidden marble. In particular, each voter will see either a white signal (denoted ω) or a black signal (denoted β). If the hidden marble is white, then each voter will privately draw exactly one signal marble from the "60W" urn. Therefore, if the hidden marble is white, each voter's signal will have a 60% chance of being white. However, if the hidden marble is black, each voter will draw their signal marble from the other urn, labeled "100B," which has 100 black and no white marbles. Each voter knows this ahead of time. Before the signals are drawn, the urns are covered insuring that no voter knows the urn from which she draws her signal marble. She simply reaches into a covered urn and draws out a marble, which is then replaced. Thus, the likelihood of receiving each private signal is conditional on the state of the world: $P(\omega|W) = 1 - P(ß|W)$; $P(\omega|B) = 1 - P(ß|B)$. The setting of our first experiment is presented in Table I; the setting of the second experiment, which assumes $P(ß|B) = .9$, is described later.

Table 1: Experimental Conditions - I	
The set of voters	N = {*i, j, k*}
The set of alternative possible states of the world (only one of which is true)	W, B
Priors	P(W) = .6, P(B) = .4
The set of possible statistically independent private signals	ω, ß
Conditional likelihoods of receiving the signals, given the state of the world[7]	$P(\omega\|W) = .6$, $P(ß\|W) = .4$, $P(\omega\|B) = 0$, $P(ß\|B) = 1$
The number of signals observed by each voter	1
The rule of group decision making	Simple majority rule voting

These conditions apply to and are known by all participants. That is, there is *common knowledge* of the structure of the game specified in Table 1.

A voting strategy is said to be *sincere* (and the voter is said to vote *sincerely*) if the voter always selects the alternative most likely to be true. Let $\phi = \omega$ or β be the private signal. By Bayes' rule:

7/ Since $P(\omega|W) > .5$, and $P(\beta|B) > .5$, the state of the world is more likely to transmit a revealing, than a deceiving signal.

$$P(B|\phi) = \frac{P(\phi|B)P(B)}{P(\phi|B)P(B) + P(\phi|W)P(W)}, \tag{1}$$

and

$$\text{Sincere vote given } \phi = \begin{cases} B \text{ if } P(B|\phi) > .5, \\ W \text{ if } P(W|\phi) > .5, \\ B \text{ or } W \text{ if } P(B|\phi) = .5. \end{cases}$$

For the parameters in Table 1, $P(W|\omega) = 1$ and $P(B|\beta) = .625$. That is, the sincere vote is to vote *w* upon observing ω, and vote *b* upon observing β. Obviously, a voter acting in solitude would vote sincerely.

A voting strategy is said to be *informative* (and the voter is said to vote *informatively*) if she votes *w* upon observing ω, and *b* upon observing β. Clearly, for the parameters in Table 1, the sincere vote is informative. Further, if all members vote informatively, then as shown below the probability that a majority is correct is greater than that of an individual:

First, consider the probability that an individual who votes informatively votes correctly. This is denoted as:

P(An individual is correct | Informative vote)

$= P(\text{vote} = w|W)P(W) + P(\text{vote} = b|B)P(B)$

$= P(\text{signal} = \omega|W)P(W) + P(\text{signal} = \beta|B)P(B)$

$= .6 \times .6 + 1 \times .4 = .76.$

Thus, a single informative voter is correct with probability .76. Now, what is the probability that a majority of 3 informative voters guess correctly?

P(A majority of three voters is correct | all vote informatively)

$= [P(\omega,\omega,\omega|W) + 3P(\omega,\omega,\beta|W)]\ P(W) + [P(\beta,\beta,\beta|B) + 3P(\omega,\beta,\beta|B)]\ P(B)$

$= .648 * .6 + 1 * .4 = .7888 > .76.$[8]

8/ Note here that we could not use the jury theorem as formulated by Condorcet because, with $P(\omega|W) = .6$ 👎 $P(\beta|B) = 1$, the votes are not independent. Instead, we use a version of the jury theorem for dependent votes (Ladha, 1995).

This aggregates via majority rule with three voters to .7888. This is a perfect example of a judgment problem, in that the members of the group have a shared preference in attaining the truth--but different beliefs based on their private signals.

Nash Equilibrium Behavior

In the experimental setting of Table 1, a majority does better than any individual **if each voter votes informatively.** But there may be at least two reasons why someone might vote *uninformatively* (where a voting strategy is *uninformative* if the vote is independent of the observed signal).

One reason is that the private signal may be insufficient to overcome the prior belief. If a voter's prior belief in W is sufficiently high, then she could vote white even after observing a black signal. In such a case, her sincere vote would be W, and it would be uninformative. In our experimental work, however, we focus on the more interesting situation where sincere vote is informative.

The second reason, offered by Austen-Smith and Banks (1995, hereafter referred to as ASB), is that it may not be a Nash equilibrium[9] for all voters to vote informatively. They argue that as a member of a group, an individual makes a difference to the outcome only when she is *pivotal*--that is, when she makes or breaks a tie. Hence, in calculating how to vote, an individual may adopt a pivotal voting strategy because if she is not pivotal it does not matter how she votes. Based on the assumption that the others vote informatively, the pivotal voter can infer the total number of ω and β signals that the others must have for there to be a tie. Note that this inference is drawn before anyone observes a signal. This, however, may lead her to vote against her private signal with potentially adverse consequences for the jury theorem.

9/ A Nash equilibrium is a set of strategy choices such that no individual has an incentive to change her choice after discovering the choices of others. The concept of a Nash equilibrium has become a benchmark for rational behavior in contexts where groups of individuals do not explicitly choose to coordinate their strategies (i.e. in non-cooperative games). One difficulty associated with the concept is that in many games there are numerous Nash equilibria. Hence, it may not rule out very much.

To illustrate, refer to Table 1 or Figure 1. Suppose voter i, acting in solitude, receives a black marble as her private signal. As noted above, P(B | β) = .625 which leads her to vote b which is an informative vote. As a member of a group, however, suppose voter i assumes that (a) she is pivotal, and (b) the others are voting informatively. By (a), one of the remaining voters must vote b, and the other w. By (b), the one voting b must have observed β, and the other voting w must have observed ω. But for any voter to observe a white signal, it must be that the hidden marble is white! Therefore, voter *i* would vote white, no matter what she observes. Specifically, the voter would vote *uninformatively*.[10] Thus, for some parameter values, including the experimental setting of Table 1, the inference about total number of ω and β leads a pivotal voter to ignore her own signal. But if everyone acts as a pivotal voter, and assumes everyone else to be informative, everyone would vote w and the advantages claimed for MR by the Condorcet jury theorem would disappear.

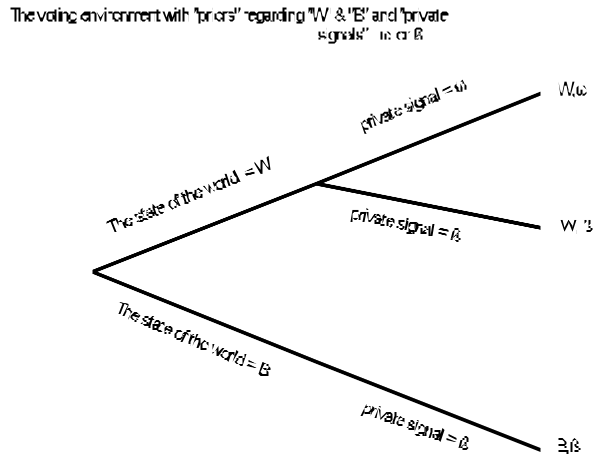

Figure 1: A simplified example showing the objection to the assumption of informative voting in the Condorcet Jury Theorem.

It follows that voter *i*'s probability of being correct will vary depending on whether she acts 1) alone and votes informatively, or 2) as a member of a jury and votes uninformatively. As shown above, P(Correct | Informative vote) = .76, and clearly, P(Correct | Uninformative vote = *w*) = P(W) = .6. How does it affect the jury theorem?

The jury theorem implicitly assumes that *any* individual's probability of being correct is the same whether she acts in solitude or as a member of a group. In footnote 5, a voter is correct with probability .6 and the same number is used to compute the group's probability of being correct. This implicit assumption is embedded in the theorem's proof. Now, each person's information enables her to be correct with

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10/ By always voting white, i improves the performance of the majority when the other voters, voting informatively, cast a split vote, and makes no difference otherwise. As an example, consider judging articles submitted to journals. If a referee believes that conditions akin to those in Table 1 apply, and that the others see one independent signal and vote informatively, then she may vote *pivotally* against the publication of the article, without even bothering to read the article.

probability .76, but a pivotal strategy changes her p_i to .6 thus violating the implicit assumption. The violation occurs because it is not a Nash equilibrium for everyone to vote informatively.

If, however, everyone votes as if she were pivotal (that is, uninformatively for a specific alternative) it **is** a Nash equilibrium. Under the conditions sketched in Table 1, such voting leads to everyone voting white regardless of their observed signals. If any **single** voter considers whether or not to vote informatively, she discovers that informative voting (or any other strategy) neither changes the outcome nor improves her payoff: With two people always voting white, the majority will still be correct 60% of the time (because the hidden marble will be white 60% of the time). So pivotal voting by all is a Nash equilibrium. This Nash equilibrium leads to decreased accuracy of group choice when compared to how a single voter would make the decision; a given voter voting her beliefs based on her signal is correct 76% of the times; but when everyone treats her vote as crucial to a good outcome, then the accuracy drops to 60%. Majority rule synergies are in this case actually negative. The group could do better by relying on any individual than by relying on majority rule. To restate the main points: Pivotal voting threatens the synergy which was calculated in the CJT. Indeed, the beneficial situation in which all vote informatively is **unstable** while the detrimental situation in which all vote uninformatively is **stable**.

How serious is this limitation to the jury theorem? Are these non-informative Nash equilibria likely to be manifest empirically? Recall that the assumptions needed for pivotal voting to be uninformative are not those of general properties of variables of interest (such as concavity of utility functions). The assumptions are about details about the probabilities of what others know. To see this, start with Table 1 but change $P(\omega|B)$ to .39 (see Appendix II for details). When $P(\omega|B) = .39$, a voter voting pivotally would vote informatively; if $P(\omega|B) = .41$, the pivotal voter would vote uninformatively. A slight change in $P(\omega|B)$ has caused a sea change in the voting behavior of the pivotal voter. Is it realistic to expect many voters to change their behaviors on the basis of the implications of such details of the probabilities of the knowledge of others? We think not. Yet, such detailed knowledge of the structure of the game is essential for voters to choose to vote as if they were pivotal. We believe that most voters are unlikely to act pivotally, even though that is one Nash equilibrium in the experimental setting described above.

Existence of Improving Equilibria

Are there other, more compelling Nash equilibria? If so, what do they imply about information aggregation under majority rule? If the structure of priors and conditional probabilities are such that by herself, each voter votes informatively, then uninformative voting arises from the individual's presumption of being pivotal. Let the parameter values be such that the pivotal voter votes uninformatively (for W say). Thus, the pivotal voter would vote *w* irrespective of her signal. We are interested in knowing if any outcome other than "all pivotal" is a Nash equilibrium, and if so, what is the accuracy of that Nash equilibrium.

By way of an example, let us compute the probability that a majority votes correctly when the parameters are as given in Table 1 and when the number of uninformative voters is 0, 1, 2 and 3. As shown in Table 2, when two or three jurors vote uninformatively for W, a majority would be correct as often as the hidden marble is white, that is, 60% of the times. When all vote informatively, as shown before, P(Majority is correct | all vote informatively) = .7888.

Finally, when there is one uninformative voter voting *w*, we have:

P(Majority is correct | one uninformative voter) = $P(\beta,\beta|B)*P(B) + [1 - P(\beta,\beta|W)]*P(W)$

$= .4 + (1 - .16) * .6 = .904.$

Thus, the accuracy of a MR group operating with exactly one uninformative voter is 90.4%, an improvement over the 78.9% with three informative voters.[11]

Table 2: Relating the Number of Uninformative Voters & the Quality of Outcomes

No. of informative Voters	No. of uninformative Voters	Probability of Correct Group Choice	Group Choice	Is the Outcome Nash?
3	0	.789		No
2	1	.904	W	Yes
1	2	.6	W	No
0	3	.6		Yes

So is there an incentive for one of the informative voters to vote uninformatively, **knowing that one other voter is already voting uninformatively?** The answer, of course, is "no." By adding a second uninformative voter to the three-person group, the group will inevitably be wrong whenever the hidden

11/ Note that given Table 1, the optimal rule is to select w unless all vote b. Under such a rule all would vote informatively. But we start with majority-rule voting as given and let the voters vote as they wish.

marble is black, that is, 40% of the time. The incentive to vote uninformatively exists only when the subject believes that the other voters are not voting uninformatively. Neither does the single uninformative voter have any reason to switch to voting on an informative basis. In other words, having one uninformative voter is also a Nash equilibrium which seems more compelling than the equilibrium at which all voters throw away their information and vote for the same alternative.

The following Proposition generalizes the findings of the above example.

Proposition: Suppose by herself each of the n jurors is informative. Let $P(\beta|B) > .5$, $P(\omega|W) > .5$, n be odd, and the signals be conditionally independent. Let it be the case that the jury theorem for dependent votes holds, but informative voting by all is not a Nash equilibrium. Then, there exists a Nash equilibrium at which

(a) a minority votes uninformatively, and

(b) the probability that a majority votes correctly exceeds that obtained under Condorcet's Jury Theorem.

Proof. See Appendix I.

The theorem states that there exists an equilibrium at which a minority votes uninformatively while a majority votes informatively. Moreover, at this equilibrium, the set of voters perform even better than what would be predicted by the jury theorem. Thus, collectively uninformative voting by a minority of voters advances the interest of all. MR information aggregation occurs as a majority of voters continues to vote informatively.

If our concern is with effective information aggregation, then *this* Nash equilibrium is of the utmost importance. In this case, a "little" pivotal voting actually improves on the optimistic results of the Condorcet jury theorem, rather than confounding them. Hence behaviorally, the Condorcet jury results may be a sort of a lower bound on the effectiveness of MR information aggregation, rather than an upper bound. The assumption of informative voting would then appear to be a conservative assumption because it underestimates the effectiveness of MR voting.

To summarize, there are Nash equilibria which improve upon the accuracy of informative majority rule, but there are other equilibria which do the opposite. The purpose of the empirical section of the paper is to find out which Nash, if any, are likely to obtain.

Nash Equilibria and the Coordination Problem

Multiple Nash equilibria (especially those which differ in value) can pose difficulties. In this case, the self-evident way to play may be to vote in such a way that the group has the benefit of your private signal. The reasons for doing so would, first of all, be cognitive. It is a cognitively daunting task to understand why it might be advantageous to the player and to the player's group to ignore a black signal and vote white all the time. It requires a rather sophisticated grasp of Bayesian probability (which we know from other psychological experiments is not descriptive of how most people make risky choices).

On top of this cognitive task, however, is an even more daunting coordination task. The coordination problem is one which Kreps describes as "too many equilibria and no way to choose." There are situations in which game players apparently have the knowledge to "solve" such problems, but as Kreps points out,

> This knowledge comes from both directly relevant past experience and a sense of how individuals act generally. And **formal mathematical game theory has said little or nothing about where these expectations come from, how and why they persist, or when and why we might expect them to arise.** (p. 101, emphasis in the original)

In other words, while perhaps votes should be in equilibrium, they need not correspond to a particular equilibrium.

There may, in fact, be situations in which no form of Nash equilibrium is the "self-evident" way to play the game. In the three person game, one has to figure out whether the other two players intend to vote informatively or not. In a seven person game, the optimal equilibrium number of pivotal voters may be any number up to three, depending on the parameters of the game. If one can figure out that exactly two voters should vote uninformatively, then there are 21 possible equilibria which are equally advantageous to the group. Of course given that the CJT shows that as N increases the probability of reaching a correct judgment increases exponentially, the gains from any pivotal voting are likely to decrease as N increases.

Meanwhile, there are cultural norms prescribing a much easier task. Our culture teaches us that it is important to vote our own beliefs; this may provide a voter with support for a decision simply to vote informatively--whether or not that constitutes a Nash equilibrium strategy.

To reiterate, there are three issues left to be resolved empirically. (1) When faced with a group judgment problem of identical preferences and private information, do any or all individuals vote as if they were pivotal? (2) If and when they do, does it lead to a Nash equilibrium? (3) If so, is the Nash equilibrium one in which group judgmental synergies are better than those predicted with informative voting, or worse? The following design is intended to clarify these issues.

Our expectation was that individuals vote informatively even as their rewards depended on group performance. Consequently, our first experiment was designed to maximize the chances of observing uninformative voting. That is, we intended to give uninformative voting its "best chance", on the assumption that in most settings uninformative voting would be even less likely. If we observed majority rule judgmental synergies even in those situations in which the likelihood of uninformative voting was maximized, then we could confidently state that judgmental synergies were not sensitive to the phenomenon of uninformative voting.

Experimental Tests

Experiment I

The experimental procedure described in Table 1 and Figure 1 maximizes the likelihood of uninformative voting by making a single white signal completely reveal the color of the hidden marble. That being the case, any voter who considers the possibility of being pivotal must realize that she should vote "white"--even after observing a black marble.

Groups of seven subjects (from Washington University) were selected at a time. Instructions were read to the subjects. Subjects were assigned a player number from one to seven. Players one, three and five constituted one decision-making group, and players two, four and six constituted another decision-making group. Subject seven was assigned the task of selecting one hidden marble for each of the two groups (with replacement), during each period, and revealing each hidden marble at the end of each period. Subjects never knew who the other two members of their group were.

As stated earlier, we induced prior beliefs in experimental subjects by showing them an urn marked "60W," containing 60 white and 40 black marbles. They are told the composition of the urn, as well as given the chance to observe it.

Each voter is given a *private* signal that is conditional upon the color of the hidden marble. If the hidden marble is white, the experimenter offers each voter a chance to draw one marble from the "60W" urn; if the hidden marble is black, each voter has a chance to draw one marble from the second urn labeled "100B."

Each voter knows this ahead of time. The urns are covered so that no voter knows the urn from which she draws her signal marble.

As discussed above, in this situation, it is not a Nash equilibrium for all three voters to vote informatively. It is, on the other hand, a Nash equilibrium for all three to vote white: no one person, by voting black can change the outcome if the other two are voting white. Thus, it may be consistent for each voter to go through the process of voting as if she were pivotal, and vote white regardless of her signal. Groups consisting of such pivotal voters will be correct only 60% of the time. This compares unfavorably with both the accuracy of each individual (76%) and the potential accuracy of groups composed of (non-Nash) informative voters (78.9%) deciding by MR. But recall there are three other Nash equilibria (see Table 2) in which only one of the three voters votes pivotally. In each of these Nash equilibria, the group accuracy would be 90.4%, rather than 78.9% if they all vote sincerely. However, achieving any of these Nash equilibrium would seem to require the solution of a coordination problem: which voter is to be the designated pivotal voter?

Because the coordination problem would seem to be critical to achieving the effective Nash equilibrium, we designed two experimental treatments. In Treatment 1, subjects were given less feedback about the behavior of others than in Treatment 2 in which subjects were given **full** feedback.

Treatment 1 consisted of six periods of choice, followed by a questionnaire administered to the subjects. The reward for each individual in a successful group was $1 in the first period, then $5, $1, $15, $1, and $10. In each case, the individual reward was earned if and only if a majority of voters in their group voted the correct color of the hidden marble. After each period, the subjects were shown the total number of white and black votes for each group, and the hidden marble was revealed. Thus, in treatment 1, they could make no simple inferences about which other voters were voting pivotally or informatively.[12]

Treatment 2 was identical to Treatment 1 with one exception. In Treatment 2, subjects were told <u>after</u> each round the color of every group member's private signal, and how each one voted, so that pivotal voting by any group member was completely apparent.

12/ But when the color was revealed as black, and when the votes were revealed, the perceptive subject could calculate if there were any pivotal votes cast.

Four groups were composed of business school freshmen, and four groups were composed of economics department graduate students, recruited from their statistics class. The subjects were divided into two treatments.

Results

The results, shown in Table 3, are a striking confirmation of the efficient information aggregation potential of majority rule. Overall, the eight three-person groups made group-based decisions in 48 different periods (neglecting the periods in Treatment 2 in which payoffs were based on individual predictions). The groups made accurate decisions 93.75% of the time.[13] This appears to be an improvement on the theoretical accuracy expected of each individual (76%), and is even an improvement over the accuracy predicted by the Condorcet jury theorem (78.9%). Most obviously, the observed majority rule accuracy is inconsistent with the Nash equilibrium in which all subjects engage in uninformative voting, which has an expected accuracy of only 60%. We now explore the reasons for this remarkable accuracy.

Table 3: Accuracy of Group Majorities (when pay was for group accuracy)

Experimental Treatment	**Color of Hidden Ball**	**Color chosen by majority**		**Group Accuracy**
		White	**Black**	
1	**White**	11	3	76.9%
	Black	0	10	100%
2	**White**	11	0	100%
	Black	0	13	100%

Recall that an individual would never vote *b* after observing ω because a white signal precludes a black hidden marble. Thus, a chance to vote uninformatively arises only after observing a black signal. It is only the behavior of subjects observing a black signal that is diagnostic of an informative or uninformative voting strategy.

In Table 4, we present the results of this experiment for both freshmen and graduate students under both low and high feedback environments summarizing the voting behavior of each player. The players of

13/ And in Treatment 2, where the voters had more information regarding the patterns of voting as a function of signals in the group, the record is even better.

odd numbered groups are represented by 1, 3, 5; those of the even numbered groups by 2, 4, 6. The row titled "# of pivotal votes" is the number of periods in which the player voted white with a black signal. The row titled "# of chances to be uninformative" is the number of periods in which the player observed a black signal. For example, in two of the six periods, player 1 of Group 1 observed a black signal and each time voted black--that is, informatively. The final row marked "Outcome" characterizes the outcome of each group for all six periods combined as follows:

Table 4: Results of the First Experiment

FRESHMEN STUDENTS	**Low Feedback**		**High Feedback**	
Group & Voter Numbers	**Group 1** **1 3 5**	**Group 2** **2 4 6**	**Group 3** **1 3 5**	**Group 4** **2 4 6**
# of pivotal votes	0 0 0	0 0 1	3 0 0	0 0 0
# of chances to be uninformative	2 2 2	3 4 5	4 3 4	3 4 2
Outcome	Informative non-Nash	Informative non-Nash[14]	Coordinated Nash	Informative Non-Nash
GRADUATE (Ph.D.) Students	**Low Feedback**		**High Feedback**	
Group & Voter Numbers	**Group 5** **1 3 5**	**Group 6** **2 4 6**	**Group 7** **1 3 5**	**Group 8** **2 4 6**
# of pivotal votes	0 0 0	4 0 0	2 0 0	0 5 0
# of chances to be uninformative	3 4 5	5 4 4	6 6 5	5 5 4
Outcome	Informative non-Nash	Coordinated Nash	Informative non-Nash	Coordinated Nash

(a) All vote informatively. Because it is not a Nash equilibrium, we call it "Informative non-Nash."

(b) A Nash equilibrium at which there is exactly one pivotal (i.e. purposefully uninformative) and two informative voters. Because actions must somehow be coordinated to attain this equilibrium, we

14/ Voter # 6 of Group 2 voted white with a black signal in period 1, but did not do so again in the four more periods in which he received a black signal. Also note that Voter # 1 of Group 7 voted white with a black signal twice, but did not do so again in the four more periods in which he received a black signal. Although classifying these voters as uninformative would be consistent with the Nash equilibrium in (b) above, we do not think of them as uninformative because they clearly fail to reflect the logic underlying uninformative voting.

designate it coordinated Nash. It is consistent both with our claim that it exists and the claim of ASB that all voters will not vote informatively.

(c) A Nash equilibrium at which all players vote pivotally (i.e. purposefully uninformative); this is the Nash equilibrium which is detrimental to MR information aggregation.

In our experiments, not a single case of the detrimental Nash equilibrium [type (c)] occurred: Never did all players of any group vote uninformatively in any period. Therefore, the empirical findings are consistent with synergistic information aggregation by MR voting. As we will see in a later section, the story repeats in the second experiment.

All groups ended at either a coordinated Nash or non-Nash equilibrium outcome. We thus have the following: Five of the eight groups exhibited the non-Nash profile: Each voter in these five groups persisted with informative voting. The behavior of these groups was thus exactly captured by the Condorcet jury theorem, and they captured the synergistic benefits predicted by the CJT.

The key to the improved group accuracy in the remaining three groups, including two with Ph.D. students, was that **exactly one person voted pivotally per group**. In two of the successful coordination cases there was with full feedback and coordination was relatively easy. The third group which experienced successful coordination did **not** enjoy full feedback. But in this case also, it was clear that it was coordination, not luck, that kept the number of pivotal voters down to the ideal number. In this group, the decision to vote *informatively* was made quite consciously by one voter who deduced that one of his fellow group members was voting pivotally, and that a second pivotal voter would be harmful. This voter wrote:

> I realized that one of the members of my group was voting "white" regardless of the actual color of the signal he received. After I realized this I knew not to deviate from choosing the color of my vote to be the same as the color of my marble.

He realized that a fellow group member was voting pivotally when a single white vote was reported after a period in which the hidden marble was black. Since all the group members necessarily received black signals, then a vote total showing one "white" voter indicated a pivotal voter, and the voter was warned off from doing so.

<u>An Equilibrium Check: Experienced Voters</u>

With the exception of self-reported information from subjects, it is difficult to tell whether or not the existence of the ideal--one pivotal voter per group--was accidental or the result of successful coordination to a Nash equilibrium. It is possible that the graduate students, whatever be their background, were so distributed that each group had one pivotal voter. If so, then if we placed all three of the known pivotal voters in a group, they might all continue to vote pivotally, producing the Nash equilibrium in which all vote w. On the other hand, if it was the result of careful coordination, then placing known pivotal voters together would just be another challenge to their powers of coordination, eventually resulting in two former pivotal voters deferring to a third.

As a check on this, we called back six of the 12 graduate student subjects. We re-ran a full feedback experiment with them. The membership of subjects in groups was once again unknown to the subjects; however, we secretly guaranteed that the three previously pivotal voters were in one group, and that the three previously informative voters were in another. After the experiment, the subjects wrote essays explaining their behavior, with a prize of $10 for the clearest exposition of their thinking.

The result was once again the ideal Nash equilibrium outcome: one subject voted pivotally in each group. What is more, the essays indicated that, by the end of the experiments, the experienced subjects, understood the advantages of exactly one pivotal voter per group. They also had an awareness of the coordination problems to be overcome to get to that outcome. Among the three pivotal voters, two switched to informative voting out of a clear recognition that two pivotal voters was one too many. One previously strategic voter stated:

> The underlying strategy for my vote when I received a black signal is that some one in my group voted white regardless of his signal, so my best vote . . . was to vote black. If the hidden marble was black then a majority, 2 of 3, would receive black signals and our group would get the reward. If the hidden marble was white then the only way we would not get the reward would be if both of us who played our signals received black signals. I am not sure how the player who played W each period decided to do that, but once he did it was clear that I should vote my signal each period.

The person who did vote pivotally did not do so out of some miscalculation: he wrote, "If payoff is based on my own vote, <u>I know to vote my signal</u>. (Emphasis his own.)" But "if payoff is based on majority vote, what is best for my group is for one player to vote W regardless of signal and the others to vote their signal." He worried about the coordination problem, but he received a black signal in the first period, and figured that the logical way to establish who was to vote pivotally was for someone with a black signal early in the game to vote pivotally. So he voted pivotally knowing that it would be visible to the others, and hoping that the others would then defer to him.

Among the three previously informative voters, they seemed to be equally aware in this replay, at least, of the advantages of having one pivotal voter. The pivotal voter in this group wrote:

> Why did I vote for [white] when I received a black signal? I knew the best strategy would be to have one player from each group always vote white regardless of his signal and I hoped the rest of my group would be able to figure this out also. So the question was, "Who should play white always?" Since I received and voted white in the first period as my two teammates voted black, I seemed to be the logical choice to always vote white.

His coordinating device, in other words was that the two people who had voted black in the first period had demonstrated their intention to vote informatively--therefore it was up to him to vote pivotally.

The evidence of this play among pivotal graduate voters was convincing to us: not only were people voting uninformatively in order to enhance group accuracy, they were also doing it in the correct proportions to achieve the advantageous Nash equilibrium, rather than the all-pivotal Nash equilibrium that is harmful to group accuracy.

What To Do in Low Feedback Environments: A Cultural Norm in Favor of Informative Voting?

In the three person experiments attaining a Nash equilibrium in which a single pivotal voter makes everyone better off creates a coordination problem. Why should any one voter presume herself to be the pivotal voter? The danger in this, as in any coordination problem, is that either everyone or no one will vote pivotally. Obviously, from the point of view of group welfare, the greater damage would stem from all voting uninformatively.

This did not occur in our first experiments, partly because of extra feedback in Treatment 2, and the potential for accurate inference in the feedback in Treatment 1. But what about other situations in which less feedback is provided--for instance, one-shot games? Achieving an asymmetrical Nash equilibrium in such a case requires a distribution of beliefs about others' presumptions and strategies that decision-makers cannot be presumed to have easily. One suggestion could be to assume that everyone else is voting sincerely. But if everyone makes that assumption and, thereby decides to vote pivotally, then everyone is worse off. Everyone would be better off by voting informatively.

In such a situation, a cultural bias in favor of informative voting would be a useful social institution, much like a social norm which tells members of a social group how to solve other coordination problems, such as which side of the street to drive on. Collective judgment problems, such as jury decisions, panels of experts, group study assignments in school, could easily be identified as those where there is a right or wrong answer to be achieved, creating a common goal which makes pivotal voting socially inappropriate. That is, citizens may approach voting in a jury situation quite differently than they would voting for their favorite candidate in the election to the Elks presidency or the city council. The latter situations is one in which preferences are at odds; the former situation is one in which there is potentially a right and wrong answer, and we may well inherit a cultural belief in a citizen's duty to vote informatively.

The subjects in our experiments seemed to feel an obligation to share information honestly. The subjects in our experiment largely agreed with the statement (Q9): "When I made my decision, I believed it was very important for me to vote the color of my own signal." Players 2 in Group 6, and 4 in Group 8, the two most frequent pivotal voters, disagreed strongly (1 on a seven point scale). Two other pivotal voters responded with two 4's, indicating ambivalence. Fully 19 players agreed with the statement, and acted on that belief in the experiments. Table 5 shows that the response to the question differed greatly by the choice of strategies of the voters.[15]

Table 5: Attitudes about voting one's signal informatively

Independent Samples T-test on Belief of Importance of Voting One's Signal
Grouped by Voter's Choice of Strategy

VOTER TYPE	N	MEAN	SD
Informative	19	6.211	1.032
Pivotal	5	3.000	1.871

POOLED VARIANCES T = 5.203 DF = 22 PROB < 0.0005

15/ The simple Pearson correlation between the two variables: the response to question 9 and the voter ever being a pivotal voter is -0.743.

The proportion of subjects who understood the importance of being able to share their private signals in a pre-voting communication period is even greater. Twenty-one of twenty four subjects agreed that they would have told other group members "promptly and accurately" about the color of their signal.

Such a cultural norm of information sharing in group judgment problems could be supported by an awareness that MR can create synergies in information aggregation. This awareness, too, might be part of our cultural heritage, stemming from elementary school days in which it dawns on us that it is unfair to answer math tests as a "group project" simply because the group is so much more effective than any one individual. Of the twelve people who were asked to agree or disagree with the statement, "I would prefer to be paid on the based on the accuracy of the group majority rather than on the accuracy of my own prediction," ten agreed and two strongly disagreed--neither of whom were pivotal voters.

Furthermore, such a norm might be more useful if it were applied more often in situations that are cognitively more difficult or strategically more complex, in which the "right" thing to do would seem to be difficult to find. Such a setting is created in Experimental Design II.

Experimental Design - II

Recall that the first experiment was designed to maximize the chances of observing pivotal voting. By insuring the receipt of a black signal whenever the hidden marble was black (i.e. P(ß|B) = 1), the subjects were not required to engage in complicated Bayesian calculations of the sort that have been found to be beyond most experimental subjects (again, see Kreps, p. 101). Rather, the subject is merely required to think of the possibility that she might be the pivotal voter: a possibility which only occurs with the existence of a white signal, and one white signal indicates a sure thing: the hidden marble can't be black.

In this sense, Design I was a limiting case designed to give the theoretical possibility of pivotal voting the greatest laboratory opportunity to demonstrate a non-intuitive, previously unsuspected effect. Under those conditions, the theoretical possibility of pivotal voting was empirically realized. With it came an unexpected turbo-charging of MR judgment accuracy. On the other hand, can one expect pivotal voting to show up under more general conditions, i.e. when there are no sure things and hence, unlike in **Figure 1**, both branches would fork?

To this end, we designed a second experiment. It is similar to Design I, except that P(ß|B) = .9 rather than 1.0. The probability of a black signal with a black hidden marble is set at 90%, instead of 100%. This small change has some huge implications. It is still the case that the Bayesian belief should be white (W) with a white signal (ω), and black (B) with a black signal (ß). It is still the case that, if one is pivotal with two other informative voters, one should vote white with a black signal. It is still the case that the optimal number of pivotal voters is exactly one, and that this constitutes a Nash equilibrium.[16]

However, with P(ß|B) = .9, a white signal could be observed with a hidden marble of <u>either</u> color, and there are no sure inferences which can be drawn from either of the signals anymore. As a result, the cognitive complexity of calculating appropriate responses is much greater. Further, the strategic problem is made more difficult: If a subject views a fellow group member voting white, when the hidden ball turned out to be black, it no longer means that that group member is definitely a pivotal voter; she could have simply voted informatively. After all she could have received a white signal, since P(ω|B) = 0.1.

Table 6: Experimental Conditions - II

The set of voters	N = {*i, j, k*}
The set of alternative possible states of the world (only one of which is true)	White, Black = W, B
Priors	P(W) = .6, P(B) = .4
The set of possible private signals	ω, ß
Conditional likelihoods of receiving the signals, given the state of the world	P(ω|W) = .6, P(ß|W) = .4 P(ω|B) = .1 P(ß|B) = .9

We ran 4 groups of this experiment with graduate students, mostly Ph.D. students in business and the social sciences, all of whom had taken classes in the theory of probability. Two experiments were run in one session with high feedback, and the other two were run simultaneously in a session with limited feedback, as described earlier.

There were 9 periods in the high feedback experiment, and 6 in the low feedback experiment. All payoffs were determined by MR accuracy.

<u>Results</u>

16/ Using these numbers, it is relatively straightforward to make calculations analogous to those reported for design I. The probability of an informative vote being correct is .72. The probability of a majority being correct, with no pivotal voters, is .809. The probability of a majority being correct, with one pivotal voter, is .828. The probability of a majority being correct, with more than one pivotal voter is .6.

In the high feedback experiments, see Table 7, we once again observed *one pivotal voter per group*. In Group 11, player 5 voted white six of the six times he received a black signal. In Group 12, player 4 voted white four of the four times he received a black signal. The other voters in these groups voted informatively.

Table 7: Results of the Second Experiment

GRADUATE (Ph.D.) Students	Low Feedback		High Feedback	
Voter Number	**Group 9** 1 3 5	**Group 10** 2 4 6	**Group 11** 1 3 5	**Group 12** 2 4 6
# of pivotal votes	0 0 0	0 2 0	0 0 6	0 4 0
# of chances to be uninformative	5 4 5	4 4 3	6 9 6	5 4 5
Outcome	Informative non-Nash	Informative non-Nash	Coordinated Nash	Coordinated Nash

Note: *Informative non-Nash*: All vote informatively; it is not a Nash equilibrium. *Coordinated Nash*: A Nash equilibrium with one pivotal and two informative voters; the coordination is required to have exactly one pivotal voter.

Under low feedback conditions groups made accurate predictions 9 out of 12 periods (75% accuracy). In two periods, a majority of signals (and votes) were black with a white hidden marble. Under high feedback conditions, groups were correct 15 out of 18 times (83% accuracy). The three incorrect times were when groups received two or three black signals with a white hidden marble. Player 5 saved his group from incorrectly diagnosing a white hidden marble on one occasion; on the other nine occasions of pivotal voting the hidden marble was black, but the two informative voters created an accurate majority in each case.

The two limited feedback experiments, on the other hand, revealed *nothing* that could be called pivotal behavior. Group 9 revealed a non-Nash profile of informative behavior. Two voters were perfectly informative, the third voted informatively in all periods but one. In this period, her vote can only be described as a mistake: She voted black with a white signal. Recall, to vote white with a black signal could be regarded as being pivotal, but to vote black with a white signal is a mistake.

Group 10 also had one subject with flawed thinking. She said that her rule was to vote white with a white signal, and every other time she had a black signal. Her explanation was that, with a black signal, there was a significant probability that the hidden marble could still be white. Her behavior did, however, generate one correct prediction for the group when a majority of voters received black signals with a white marble.

This experimental design is especially challenging because of the difficulty in the cognitive task facing the subjects, and because of the limits on the feedback about other subjects' behavior. In this case, there seems to be no systematic pivotal voting at all, even with highly trained Ph.D. students. Two of the students made flawed inferences; but even the other four had no basis for voting pivotally, because pivotal voting <u>requires a confidence that other voters are voting informatively</u>.

With no basis for solving the coordination problem, the default behavior is evidently sincere voting (with mistakes). The interesting point is that, even with some inferential flaws, the benefits described by the CJT seem to have been largely realized. Overall, in seven out of twelve groups, we have what is best characterized as informative voting by all subjects. The result is the non-Nash outcome: all vote their signal. In the remaining five groups, not all subjects vote informatively, but the outcome corresponds to the optimal Nash equilibria we identify: There is exactly one pivotal voter with two informative voters. Hence, the information aggregation attained in these five groups is even better than that attained as per the CJT. Finally, there was not a single case of the "bad" equilibria. In the low-feedback experiments, 24/36 or 67% of the individuals got signals that matched the actual hidden marble; but 9/12 or 75% of the majority votes were correct. Thus, even in the most difficult case, the use of a simple heuristic--vote your signal--guaranteed that the benefits predicted by the CJT were realized.

Conclusion

These first controlled laboratory experiments on majority rule judgments indicate that the benefits of majority rule are robust. Groups do better than individuals, **even in experiments that were designed to maximize the advantages of uninformative voting by individuals**.

In situations of unanimous preferences and disparate information states, the Condorcet Jury Theorem appears to provide a lower limit on the accuracy of aggregating judgments regarding epistemic issues. The experiments show that, when feedback about voting makes coordination possible, subjects may coordinate on Nash equilibrium in which the groups do even better than predicted by the CJT. And even when such coordination is not possible, people do not behave in a way that is consistent with Nash at all; instead, they rely on the simple heuristic of informative voting. The implications for mechanisms to aggregate expert opinion are obvious.

While these results should be interpreted as tentatively as one would any first experiments,[17] they should be of interest to those who are trying to understand the persistence of using majoritarian aggregation rules to sum up judgements on panels to make decisions. Despite its flaws as a method of aggregating preferences, MR can be used to make judgments that improve on individual decision-making.[18]

These positive results regarding aggregate panel decision-making is still the certitude of minimal competence by individual voters. If the individual facing a binary choice is less competent than a flip of the coin, then MR magnifies the error. E. B. White defined democracy as "the recurrent suspicion that more than half of the people are right more than half of the time." When this suspicion is warranted, MR can radically magnify the individual's competence.

These theoretical and empirical results support a conclusion not unlike the major theorem in economics:

> Majoritarian, collective judgements improve upon the quality of outcomes which can be counted upon by individualistic decision making. This is so when the decisions are decentralized, and based on private information, much as is the case with market outcomes.

In our time and country, when virtually all the arguments are against the experts and the planners, and pro the outcome of markets, it is useful to consider that democratic methods can allow regulatory panels to have solid tendencies to reach useful outcomes by improving upon the decisions of single decision makers.

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17/ We know of no other experiments regarding the Condorcet Jury Theorem.

18/ The experiments also have strong implications for some more general use of MR in political decision making.

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Appendicies and Experimental Protocols

Appendix I

Proof of the Proposition. Let p denote the probability that a juror's vote is correct when she votes informatively. Then:

$p = q_\omega \pi + q_\beta(1-\pi)$, where $q_\beta = P(\beta|B)$, $q_\omega = P(\omega|W)$, and $\pi = P(W)$.

Define for r = 1,...,n:

T(r) = P(a majority of the jury is correct, when r jurors vote uninformatively for *w*, and n-r jurors vote informatively).

Clearly, the accuracy of the voters, when a majority or more votes uninformatively is just π, that is, $T(r \geq m) = \pi$, where $m = (n+1)/2$. The accuracy, if all vote informatively is T(0) = P(a majority is correct as per the jury theorem for dependent votes).

$$\text{Let } T(r^*) = \max_{1 \leq r \leq m-1} T(r) \text{ for some } r = r^*.$$

We will show that $T(r^*)$, which exists because n is finite, corresponds to a Nash equilibrium. By definition, $T(r^*-1)$ could not be greater than $T(r^*)$, hence none of the r^* uninformative voters has an incentive to switch to become informative. Would any of the $(n-r^*)$ informative voters switch to become uninformative? We consider two cases.

Case (a) Let $r^* < m-1$. By definition, $T(r^*+1)$ could not be greater than $T(r^*)$, thus, none of the $(n-r^*)$ informative voters has an incentive to switch to become uninformative. Therefore, $T(r^*)$ would correspond to a Nash equilibrium.

Case (b) Let $r^* = m-1$. Then, $T(r^*)$ will correspond to a Nash equilibrium if $T(m-1) > \pi = T(r)$ for all $r \geq m$, that is, if none of the $(n-r^*)$ informative voters has an incentive to switch to become uninformative. The proof consists of a series of inequalities:

$$T(r^*=m-1) \geq T(1) > T(0) > p > \pi.$$

The first inequality follows from the definition of $T(r^*)$. To prove the second, we proceed as follows. If some voter j elects to vote either informatively or uninformatively, it would be for her own good, and by the assumption of a common goal, for the good of the group. By assumption, informative voting by all is not Nash. That is, if all except j vote informatively, then j would make herself better off by voting uninformatively. But the only way j could be better off in a dichotomous choice situation is if her voting uninformatively increases the probability of a majority being correct. That is, if informative voting by all is not Nash, then it must be the case that T(1) > T(0).

The third inequality, T(0) > p, is a consequence of the jury theorem for dependent votes. It remains to show that $p > \pi$. Because by herself each juror's sincere vote is informative, it follows that $P(B|\beta) > P(W|\beta)$, that is upon observing β each juror votes for the more likely alternative B than the less likely alternative W. By Bayes' rule, $P(B|\beta) > P(W|\beta)$ implies that $q_\beta (1-\pi) > (1 - q_w) \pi$, that is, $q_\beta (1-\pi) + q_w \pi = p > \pi$.

Therefore, $T(r^* = m-1) > \pi$. Hence, T(m-1), with m-1 uninformative voters, corresponds to a Nash equilibrium. The proof is complete.

Appendix II

Sometimes it is a Nash equilibrium for all voters to vote informatively, and then of course the claims made about the judgmental benefits of majority rule apply. To see this, change $P(\beta|B)$ to .61 from 1 in Table 1; thus, $P(\omega|B) = .39$.

As before, if the pivotal voter, *i*, observes ω, then either {ω, ω, β} or {ω, β, ω} must occur for her vote to affect the outcome. Similarly, if *i* observes β, she would presume {β, ω, β} or {β, β, ω}. Define events

$$E_\omega = \{\omega, \omega, \beta\} \cup \{\omega, \beta, \omega\} \text{ and } E_\beta = \{\beta, \omega, \beta\} \cup \{\beta, \beta, \omega\}.$$

Event E_ω applies when the pivotal voter observes ω, and E_β applies when the pivotal voter observes β. By Bayes' rule and independence of signals:

$$P(W|E_\omega) = .6995 > .5; \; P(W|E_\beta) = .4980 < .5.$$ [19]

Hence, the pivotal voter would vote informatively: *w* upon observing ω, and *b* upon observing β. This is displayed as Condition I in Table 8. Thus, under these circumstances: i.e. for P(β|B) = .61, informative voting by all is a Nash equilibrium. That is, the jury theorem and Nash equilibrium are mutually compatible. However, a small change in P(β|B) from .61 to .59, would cause the pivotal voter to switch from being informative to uninformative (see Condition U of Table 8).

Table 8: Contrasting Illustrative Conditions Showing when Pivotal Voting is Informative & Uninformative

	Condition I	Condition U
P(ω|B)	.39	.41
P(β|B)	.61	.59
$P(W\|E_\omega)$	.6995	.6853
$P(W\|E_\beta)$	.4980	.5022
Pivotal voter's decision	Vote informatively	Vote uninformatively for W

Experimental Protocols

INSTRUCTIONS

This is an experiment on group decision-making. The instructions are simple, and if you follow them carefully you could make a significant amount of money.

You are seated in a room with 6 voters, including yourself. The six voters are divided into two groups of 3 voters. Your player card identifies whether you are in Group I or Group II. The other members of your group will remain unknown to you.

In front of you are two urns. One urn is marked "60 White". It contains 60 white marbles and 40 black marbles. In a short while, the seventh volunteer will draw a marble will be one marble from this urn for Group I. The experimenter will note the color of the marble before it is sealed in an envelope. Then a replacement marble of the same color will be added to the urn, and another marble will be selected from the same urn for Group II. The problem for your group of three voters is to predict the color of that marble correctly by majority rule. If your group predicts the color of that marble correctly, you and every other member of the group will receive a financial reward. You will earn the reward whether or not you as an individual voted correctly. People who vote correctly will NOT receive the reward, if the majority of their group is incorrect. Thus, a "successful group" is one that has a two or more votes cast correctly by the members of that group.

19/ $P(W|E_\omega) = 2*P(\omega,\omega,\beta|W)\,P(W)/2*[P(\omega,\omega,\beta|W)\,P(W) + P(\omega,\omega,\beta|B)\,P(B)]$
$= (.6x.6x.4)(.6)/[(.6x.6x.4)(.6) + (.39x.39x.61)(.4)] = .6995 > .5.$
Also, $P(W|E_\beta) = 2*P(\beta,\omega,\beta|W)\,P(W)/2*[P(\beta,\omega,\beta|W)\,P(W) + P(\beta,\omega,\beta|B)\,P(B)]$
$= (.4x.6x.4)(.6)/[(.4x.6x.4)(.6) + (.61x.39x.61)(.4)] = .4980 < .5.$

Your group will be given five opportunities to earn a reward in this way, in each of five different periods. In the first period, each member of each successful group will earn $1. The possible earnings in subsequent periods will be announced at the beginning of each period, but will in some cases be substantially more than $1. You are guaranteed a minimum of $3.

You should feel free to try to earn as much as possible from this exercise. You will receive a signal before each vote. Thus, you can calculate your best vote on the basis of these instructions and on the basis of the signal you will receive. Each of you will receive your signal privately in the following way:

If your group's hidden marble is white marble, each member of your group will be given the opportunity to draw one marble from the Urn that is now marked "60 White". Thus, if your group is to predict a white marble, you will have a 60% chance of getting a white marble signal.

If the hidden marble that your group is supposed to identify is in fact a black marble, each member of your group will be given the opportunity to draw one marble from the urn that you now see before you that is marked "100 black". Thus, if your group is to predict a black marble, you will have a 100% chance of getting a black marble signal.

You could of course tell which color your hidden marble is from the urn that is presented to you; so the two urns will be hidden in tow identical pitchers, and held above your head. You will have the opportunity to reach in one of the two urns and select exactly one marble, examine it, and then return it to urn from which you drew it. After you have examined the marble, you will have the opportunity to fill out your ballot. You should make sure your player number, the period number, and either "white" or "black" is written on your ballot.

After everyone has had an opportunity to receive a signal, then the ballots will be collected, sorted by group, and the majority votes tabulated for each group. The sealed envelopes will then be opened to reveal the color of the hidden marble for each group. If two or more members of a group have predicted their group's marble correctly, then each member of that group will have earned a reward. The results will be transferred to a large board on which you will be able to see the votes from your group, the color of the marble, and the resulting reward for the members of each of the two groups. At the end of each period, if your group was successful, you should write down on your "Record of Earnings" the size of the reward that you earned for that period. At the end of the five periods, we will ask you to fill out a short questionnaire about this exercise. You will total your earnings and receive the total payoff in cash.

Player 7 will receive $15 for selecting the marbles for each group for each period, and revealing the hidden marbles at the end of each period.

This should become clearer after the first period is played. But for now, are there any questions?