JUSTICE, PREFERENCES AND THE ARROW PROBLEM[a]

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ABSTRACT

Arrow showed that there is no general way to aggregate non-interpersonally comparable preferences or welfare into either a sensible social choice or a social welfare measure. With majority rule the problem manifests itself as voting cycles. The standard response to this problem has been developing 'spatial models' built on restricted preferences (or welfare). We develop an alternative family of solutions. By assuming a culturally accepted conception of justice, we establish the possibility of sensible aggregate choice implementable via majority rule. Various assumptions regarding the form of such a utility function are discussed. Conditions for a Condorcet winner in a problem of pure redistribution are derived for a number of models. Some of the implications of this perspective for the theory of democracy are considered. Developing a normatively interesting social welfare function may require introducing normative concerns into the preferences of the individuals rather than just into the properties of the aggregation system.

Equations

JUSTICE, PREFERENCES AND THE ARROW PROBLEM

We demonstrate that it is possible to overcome preference cycles in the aggregation of voters' preferences by focusing on conditions for consensus on the nature of social justice. This offers a new solution to the instability problem that bedevils a normative evaluation of democratic choice (Riker, 1982 and Mackie, 2003). Given experimental and other evidence that there are universally constrained notions of justice among human beings, this result gives considerable weight to justice as a measure of social welfare and as an evaluative scale for political institutions.

Experimentalists have established that non self-interest and a concern for justice are general attributes of individual choice behavior. A variety of micro models of behavior have been developed to incorporate these findings (Cain, 1998; Charness and Rabin, 2000; Cox, et al., 2001; Fehr and Schmidt, 1999; Frohlich, Oppenheimer, and Kurki, 2004, Rabin, 2003). But the implications of these preferences for our understanding of democracy have not been developed (Wittman, 2002, is a notable exception). Moreover, little attention has been paid as to how our evaluation of democratic outcomes might be changed if citizens have other-regarding preferences. By focusing on a special case of other-regarding preferences, a distaste for injustice, this paper addresses some of those macro questions.

INTRODUCTION

Any proposed enterprise of this sort is immediately drawn to the seminal work of Arrow (1963). His general possibility theorem both changed the terms of the argument, and defined the grounds for examining democratic institutions. Starting with the premise that social welfare consists of the aggregation of individual welfare, he demonstrated that there is no reasonable mechanism to aggregate individual welfare (or choices) into a result that is acceptably related to

overall social welfare unless one accepts interpersonal comparisons of welfare.[1] In other words, it

is impossible to define a social welfare function (SWF) that satisfies minimal criteria for

acceptability. His method of proof was to show that aggregating individual preferences, while

satisfying a minimally acceptable set of criteria, cannot avoid cyclical group preferences. His proof

established that cycles can only be avoided by giving up desirable qualities of democratic

procedures or by restricting the diversity of preferences.[2]

By using social justice values in modeling democratic decisions we will be restricting

preferences (or violating Arrow's condition of Universal Domain)[3] to show that when individual

preferences include a conception of justice held sufficiently strongly, the problem of cycles will be

avoided. Placing a value on social justice introduces other-regarding preferences without stepping

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1. A fine discussion of the social choice literature can be had in Sen's Nobel Laureate acceptance speech. A version
of the speech has been included in Sen (2002).

2. The classical restriction of preferences, of course, has been the unidimensional 'single peaked preference'
assumption first utilized by Black (1958). Goldfinger (2002) makes an appeal to normative democratic theorists to
pay attention, but few political philosophers concerned with democracy have taken up the challenge. The very large
literature of social choice (centered on the concerns of Arrow) is quite separable from the works of these normative
democratic theorists. This in spite of the fact that Arrow has strong implications for normative democratic theory
(see Mackie, 2003).

3. Sen, and others (see Sen, 1972, especially chapters 10 and 10*), have generalized unidimensional single peaked
preferences to properties of the preferences held by the set of voters such that one does not have cycles over any
triple of feasible outcomes. More specifically, Value Restriction (VR) (p. 169) is defined so that for any triple of
alternatives there is agreement that at least one element is either not best, not worst, or not in the middle. Our model
can be related to the properties used by Sen (VR, extremal restriction and limited agreement) to develop his theorems
regarding the possibility of social decision functions. But our approach here is less abstract and formal. Rather than
identifying the particular properties of sets of preferences that permit social choice without cycles, we concentrate on
the properties of consensus regarding a substantive problem: that of distributive justice.

over the line of inter-personal comparability of utility. After developing the general approach, we present a number of models that illustrate the viability of a variety of majority rule type regimes as a mechanism for sensible aggregate choice when there is consensus on what constitutes justice and preferences regarding justice are sufficiently strong.[4]

Some might view assumptions of consensus regarding justice and relatively strong preferences to achieve it as heroic. To the skeptic, we point to a burgeoning literature of empirical and theoretical findings. Juxtaposing consensual and confrontational democracies, in research that has spanned more than 30 years, Lijphart (1999) has shown that these two different styles of democracy lead to quite different outcomes. He shows that consensual based institutional systems do better for their citizens along a number of dimensions including economic performance and social justice. Institutions alone can, of course, never guarantee consensus, but apparently procedures of public deliberation are related to the development of consensus.[5]

Also pertinent, experimental work has shown that a surprising degree of consensus exists regarding what is considered to be just in income distributions. Most of this consensus is cross cultural: experiments involving deliberation and a 'veil of ignorance' in a wide variety of cultures

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4. One could note that there are alternative regimes that can be used to aggregate votes to avoid the negative implications of most cycles (e.g. Borda count). Of course, this violates another of Arrow's conditions, and raises other normative concerns. But this is not the main concern of the paper. Rather, our concern is to show that by tapping what is likely to be a preexisting underlying consensus regarding justice, democracies have a way of solving a number of related problems. These include tethering social welfare to democratic outcomes, overcoming cycles, or related arbitrary outcomes of political processes.

5. Numerous philosophically oriented democratic theorists have written of this. See for example, Fishkin and Laslett, 2003 for a solid collection of essays on the subject. Also see Dryzek and List, 2002; Knight and Johnson, 1994; Fung and Wright, eds., 2003; and Guttman and Thompson, 2002.

(Cruz-Doña & Martina, 2000 - Philippines; Frohlich and Oppenheimer, 1992 - Canada, USA, and Poland; Jackson, 1995 - Australia; Konow, 1996 & 2000 - USA; Oleson, 1997 - USA; Saijo et. al. 1996 - Japan) showed similar predispositions, choices, and attitudes. In all cases, individuals agreed that justice has three competing normative components: need, entitlements and efficiency.

Miller (1991) was one of the first to tie deliberation to the social problems with which we are concerned. He noted that public deliberation worked in numerous ways to develop consensus regarding the nature of the political issue under consideration. Public deliberation cuts down on the sorts of arguments that can be made for and against options, motivating generalized evaluative concerns rather than narrow self-interested ones. It also tends to weed out the use of arguments based on false beliefs and preferences that are repugnant to public moral beliefs. In other words, Miller argued that deliberation transforms some preferences, and results in a smaller, less diverse, set of preferences to be aggregated. Formally, he ties the argument to the development of a consensus regarding the dimensionality of political choice problems and solving the choice one dimension at a time.

We believe that Miller and the theorists mentioned previously are on the right track. Public deliberation helps to crystallize and bring out consensus, and hence we support their conjecture that it will help to decrease the social choice problem. But as Dryzek and List emphasize, and Miller acknowledged from the beginning, the relations between deliberation and the development of single peaked preferences may be problematic. Again our approach is at one time less abstract and more substantive - we do not try to tie consensus to all problems, but rather explore the consequences of its existence regarding social justice, and how this might affect democratic choice under majority rule, and under other institutions. Moreover, our approach is not built on single peaked preferences, but rather on agreement regarding a justice vector.

We examine the effects of consensus on a conception of justice on the relationship between individual choices and outcomes, first, in a particular institutional and policy context: majority rule decision making (MR). The ensuing preference restriction leads to a Condorcet winner even in a problem of 'pure redistribution.'[6] We then generalize the model to other decision rules and examine the implications of a shared sense of justice in those contexts.

CYCLES IN REDISTRIBUTIVE DECISIONS: AN EXAMPLE

A political institution, such as MR places restrictions on which alternatives can beat others. Under MR, constructing a winning coalition for an alternative (say to the status quo) requires a majority of the voters who prefer it to the status quo. Hence, to consider the stability of a victory of any alternative we need to consider only those alternatives that can beat it (i.e. its *winset*).[7]

To illustrate the approach, we begin by considering the archetypical cyclical majority situation: redistribution with pure self-interest. But even restricting our focus to such distributional concerns, one should note this sort of example covers cases such as how to

Table 1: An Illustrative Majoritarian Divide the Dollar Situation			
Payoffs to participant			**This proposal beats the preceding via the coalition:**
i	*j*	*K*	
1/3	1/3	1/3	(none preceding)
½	½	0	*i & j*
0	0.7	0.3	*j & k*
1/3	1/3	1/3	*i & k*

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6. Redistributive decisions are central to the problem of distributive justice and have often been used to establish the possibility of cyclic outcomes (see Shepsle and Bonchek, 1997: 57+; also note that Wittman, 2002 focuses on redistribution to analyze how non self-interested motives might limit cycles).

7. A stable winner is an alternative with a winset that is empty (i.e. no other alternative can get majority support against it).

distribute tax burdens for funding public goods or governmental programs more generally. We will then show how the preferences that include a concern for justice might resolve such problems.[8]

The problem has often been illustrated, but at the risk of redundancy, consider the inherent instability of any majoritarian outcome in a simple, 3 person, divide the dollar game when preferences are strictly a function of one's own payoffs (as shown in Table 1).[9]

From the table it is clear that using MR each of the options can be defeated: i.e. with MR none of the items has an empty winset. This is apparent by considering how any one member of a winning coalition can be bribed by the loser into a new coalition, and how this process can become cyclic. For example, imagine the status quo, or current proposal is egalitarian: each of the three individuals receiving 1/3. Then a majority can be put together to defeat the egalitarian status quo as is illustrated by the 1ˢᵗ and 2ⁿᵈ lines of the table. There two participants, i & j, come together to redistribute to themselves the 1/3 that was going to participant k so that they each share half of the dollar. That redistribution could itself be defeated by another division as illustrated by line three. Finally, that distribution could be defeated by the original egalitarian one. Since the support for any distributive proposal is strictly a function of the individual's payoff (very similar to self-interest as an assumption), there is no stability.

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8. It is noteworthy that introducing a concern for justice, inasmuch as it entails non-separable preferences, transforms the problem into a variable sum game.

9. The theoretical problem is actually broader than this as there is no core in any 'essential,' zero sum game. Zero sum implies that there is no sharing of values: one person's gain is another's loss. An essential game is one in which it pays to form coalitions. These two properties imply cyclic outcomes with any purely redistributional political issue. Majority rule is just an illustrative case for any political decision structure being employed. See Luce and Raiffa, (1957) Chapter 8.

JUSTICE & REDISTRIBUTIVE CYCLES: CUTTING THE GORDIAN KNOT

But how do self-interest, other-regarding preferences, and justice relate to the problem of cycles? Consider again the example in Table 1. Many forms of interdependent preferences, or other-regarding -behavior functions, could create stability regarding distributive decisions. For example, if everyone is angelically benevolent and values the welfare of each person equal to that of themselves, there will be no cyclic outcome (Frohlich, 1974). Donald Wittman (2002) has shown that if all (or even a minority subset) of the individuals are sufficiently averse to inequalities, they can find stability in an egalitarian outcome. For example, if they all value avoiding a marginal increase in inequality more than they value a marginal increase in their own payoff, they would stick with the egalitarian outcome. Wittman has established a somewhat stronger result: Were everyone to value a change in the average (absolute value) of the deviation from the mean income at just a bit less than a change in the value of their own payoff, the egalitarian outcome would stabilize.[10] Of course, if they mind the reduction of the deviation from equality more than they value their own payoff, it is easy to see how equality would be the majority rule outcome: but this may be asking a bit much. So, although his slightly weaker requirement may not appear totally surprising, it is a significant first step.

Although Wittman establishes that one can overcome cyclic outcomes with preferences that are sufficiently other-regarding, his formulation of the problem has two weaknesses. First, it focuses on an egalitarian motive that we question and which is not supported by the experimental data cited above. Second, it requires a level of other-regarding concerns that is quite high.[11]

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10. More precisely, Wittman shows that if each of *n* participants have similar preferences and value any change in equality $n/(n+1)$ times the change in their private payoff, equality will be a stable outcome.

11. Wittman, however, believes 'when there are many others, the level of concern does not seem all that high.' So, in

We establish that there are other-regarding preference structures that can be used in models to generate similar consequences without some of these unrealistic properties. In particular, similar results can be shown to hold if we develop a utility function that includes a more general concern than equality: a concern for justice. Further, we show that the general approach to the problem holds even when there is a considerable variance in individuals' attitudes towards justice.

JUST AND UNJUST INCOME DIFFERENTIALS

Not all inequalities need be viewed as 'bad.' Consider the parable of the ant and the grasshopper. The ant works hard and saves, while the grasshopper fiddles. When the weather turns bad, the grasshopper has no savings but the ants have saved for a rainy day. Moral:

When someone has worked hard and received 'just deserts', the difference may be viewed as positive: for example, a vehicle by which prosperity is insured.

This notion, that inequalities may be viewed positively, is reflected in data from dictator experiments with production. There, the dictator often distributes income generated by herself and a paired other in proportion to some notions of 'just deserts' (Frohlich, Oppenheimer and Kurki, 2004). We wish to harness this wider set of concerns for justice rather than a narrow distaste for inequality.

To develop our perspective, it is useful to distinguish between 'justified' and 'unjustified' income and deviations. Ill gotten gains by a neighbor (say by illicitly importing heroin) may increase general prosperity but also may not be given the same status as 'legitimate income.' More

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a very large group, it might be quite reasonable for someone to give up $1 if everyone else's income went up $2. But, of course, this is not the typical logic of transfer payments. They typically work by one person giving up $1 and it being shared by n others, each of whom receives, on average, $1/n.

particularly, unjustified income may be stigmatized. Indeed, results from ultimatum games[12] (see Bolton, 1991; Bolton and Zwick, 1995; Larrick and Blount, 1997 as well as Roth, 1995) have shown that individuals are willing to take significant losses rather allow proposers to take unfair advantage of their powers to set the terms of the division.[13] Similarly, experiments on common pool resources have demonstrated that when costly punishment of cheaters is an option (Ostrom, et. al. 1992), some individuals are willing to bear the costs of inflicting that punishment. In other words, other-regardingness has a 'moral' component.[14] Some inequalities may be good and others bad.

To capture this more complex set of moral structures requires that we consider not simply equality, but justice, broadly construed. We posit that individuals are upset by injustices and experience them as costs (in terms of lower utility). Of course, in some particular cases we could imagine a society motivated by a notion of justice founded on 'equality'. But as we indicated above, in general, we would expect justice to be a concept which covers a broader set of concerns. Still, if justice is a concern, shortfalls from justice - or the differences between the actual and the just distribution - ought to be motivators of human choice and behavior. If so, in keeping with the rationality approach, these concerns should be able to be represented in utility functions in order to model human behavior successfully.

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12. Ultimatum games can be described as follows. Two parties participate to split a sum of money: a proposer and an acceptor. The proposer offers a split and the acceptor either accepts this, or rejects it: in which case no one gets anything. The Nash equilibrium is for the proposer to offer only a tiny fraction of the money to the acceptor, who is then faced with that tiny reward, or nothing. The actual outcomes depend quite heavily on context and are usually much more favorable to the acceptor.

13. Most recently this has been tied to neurological data (Postrel, 2003).

14. Frohlich and Oppenheimer (2001) discuss plausible conditions of any moral point of view.

To begin, assume that there is some distribution of income (J), expressed as a vector over all individuals, that is considered to be 'just.' The justice vector J embodies the conception of justice shared within the society. Call each individual's income in that distribution j_i. Let Y be the vector of actual income and each individual's actual income be y_i. Then the difference between the individual's just and actual income can be characterized as the gap in justice; call it u_i. In other words, $u_i = y_i - j_i$. The vector U is the vector of justice gaps for all members of the group in question. While we are convinced that there are a number of normative elements of justice shared by a broad spectrum of humanity, we leave aside, for this paper, the details of what might cause some income to be viewed as either just or unjust. That may vary from culture to culture and from context to context, and could furnish the substance for another work. Here we assume that the notion of injustice is derivative of a sense of justice. The conception of justice is presumed to be societally shared.[15] Such a sharing can be thought of as improving the informational basis for a social choice.[16] It implies that we have both an understanding of shortfalls and some yardstick for measuring them. Rather than just aggregating preferences, we are infusing preferences with further information that shapes the ordering. We conjecture that this will improve the social performance of democratic systems. With any particular conception of justice as a given, we set out to demonstrate the relation between preferences for justice and stable social choice outcomes

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15. Such a statement regarding consensus covers a lot of territory. To do the work in our arguments J requires that there is an agreement to both the principles underlying justice and the facts underlying the justice or injustice of particular incomes. And by agreement to the principles underlying justice, we mean that there need be agreement re the trade offs between underlying segments of the concept (e.g need, desert, efficiency).

16. Sen's (2002) approach to the social choice problem is one that emphasizes the informational depravity of employing only the ordinalist preference assumptions without interpersonal comparisons. As he points out, this implies we can't get beyond Pareto optimality in normative reasoning and hence has nothing to say about conflicting interests or distributional justice.

given a majority rule decision procedure.

DIFFERING PERSPECTIVES

By assuming that the conception of justice is 'societally shared' we do not mean that there is a homogeneous *evaluation* of injustices. Further, we don't assume that individuals evaluate injustice using similar perspectives. All we assume is that injustices are commonly classified or identifiable, and that they are universally considered 'bads.'

The idea is similar to any other item that could enter into one's preferences. For example, consider 'pizzas.' It is presumed that everyone can identify a pizza. Now assume that everyone likes pizzas. Still some may be pizza aficionados, and others find pizza just OK. Some altruists really want everyone to have a good pizza tonight. Others could care less about everyone else. Some are envious when they don't get as much pizza as others, and others only care about their own portion of pizza. That is similar to how we will be handling injustices. Some may value decreasing injustices a lot: others less. Some egoists may be concerned about the personal injustices they suffer, others, altruists, may be concerned about the injustices everyone suffers. Some may be concerned only about the absolute levels of injustice, while others, relativists, may be primarily concerned about those who suffer more than average injustices. All of these differences should be able to be taken into account and analyzed.

DIFFERING RULES, BYSTANDERS

Further, below, we develop our results beyond simple majority rule. We extend the results to cover other, more restrictive, voting rules and consider situations where some individuals, for what ever reasons, don't get involved. In this manner, we hope that the approach will help develop the political implications of the phenomena of consensus in general, and allow one to

develop new ways to consider the centrality of mechanisms for the achievement of justice in politics.

In this manner our aim is not to focus on cycles, but rather to put front and center the relative achievability of justice by democracies. After all, there are other means to get to acyclicity. The hope is that by demonstrating the relationship between the patterns of justice preferences and the democratic attainment of justice, one can more reasonably judge alternative democratic structures by their ability to achieve justice for their citizenry.

DEVELOPING THE MODELS

Consider the utility function of the individual, differentiated, as indicate above, by the categories of just and unjust income. An individual's received income y_i enters as it does in a standard utility function. But what about unjust income? Although the society is presumed to have a common sense of justice J, we make no homogenizing assumption about how individuals evaluate the vector U of deviations from J. Rather, we assume that each individual evaluates U via a personal functional weighting, $f_i(U)$. All that we require is that $f_i(U)$ be monotonic, increasing with increasing injustice, and that it enter in the individual's welfare as a cost. In other words, a sense of injustice engenders a cost when injustice is perceived, and the greater the injustice, the greater is the cost. We also allow for heterogeneity of preferences across individuals, even when they evaluate injustice via the same function. An individual's weighting of injustice contains a factor α_i so that the individual's welfare evaluation of U is the product: $\alpha_i f_i(U)$.[17] Thus, the generalized functional form for evaluating one's position in any given income distribution is:

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17. This term is a more general substitute for that used by Wittman (2002) to reflect general desire for equality.

$$V_i(y_i, \boldsymbol{U}) = y_i - \alpha_i f_i(\boldsymbol{U}) \qquad \textit{Individual utility with a concern for justice } \textbf{(1)}$$

As noted above, this formulation avoids the critical question: "what constitutes justice?" To make the argument more general, we make no particular assertion about this substantive question here but rather assume that conceptions of justice can vary.[18] Yet within a given society, there may well be a consensus on the constituent elements of justice. As will become clearer, this is an important assumption in the model, and, we would argue, an important element in any endogenous societal judgment of political performance. It is a strong but, as indicated above, perhaps plausible assumption. In any case, assume there is a shared sense of what constitutes a just income distribution. It is this consensus which we assume to be captured in the common vector of just income distribution: $\boldsymbol{J}$.

Whether this conception constitutes a feasible way of stabilizing democratic decisions depends on whether it identifies reasonable valuation functions for individuals. To see whether this is the case, we begin by outlining the argument regarding stability in a majority rule context and adding some substantive content to the evaluation function f_i.

Suppose we begin with a 'just' status quo in a majority rule democracy from which redistribution would generate injustice. For the status quo to be stable no majority must be able to obtain positive individual net benefits from a redistribution of assets from a minority. In other words, a member of any majority must be sufficiently bothered by the injustice of a redistribution to turn down any ill-gotten gains. To see how this would work, imagine ordering the population by the strength of each person's distaste for injustice: $\alpha_i f_i$. For the status quo to be stable, the

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18. As indicated above, we actually hold a different view: there is a common conception of justice, and the structure of the political system, to a large extent, determines its achievement. We consider this in the discussion section of the paper.

median voter (median in the sense of the size of $\alpha_i f_i$) must find that the marginal cost of the injustice $\alpha_{i\,med} f_{i\,med}$ outweighs the marginal gain of the income $y_{i\,med}$ that she could get. We can discover the implications of this by putting some substantive content into this abstract formulation with some illustrative models with different f_i s.[19]

Since we are interested in the *political* stability of the income distribution, we first consider the possibility of a majoritarian coalition taking ϵ from each of the members of the minority starting from a just distribution *J*. Of course, the potential victims would always prefer the status quo. Hence the questions to be answered becomes: "What are the conditions that make it too costly for those receiving the unjust transfers to accept the transfers?" and "Are these conditions reasonable enough to offer hope that they might apply in the real world?"

With majority rule, a winning coalition would constitute a majority, and in zero-sum games (those with redistributions) the biggest monetary per capita payoff comes when the winning coalition is of minimal size and all others are slated as victims. We begin by presuming the $(n-1)/2$ losers share equally[20] in the loss: they each lose ϵ to a coalition of minimal size: $(n+1)/2$. Under these conditions each "winner" gets a $2\epsilon/(n+1)$ share of each ϵ for a total monetary gain of $[2\epsilon/(n+1)]\,[(n-1)/2\,]$.[21] This simplifies to

$\epsilon(n-1)/(n+1)$ *Winner's unjust gains in minimal winning coalition from unjust transfer* **(2)**

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19. Implicit in this approach is the notion that the f_i, by including concerns for others' welfare or status transforms the 'purely redistributive' (or zero sum) problem into a non-zero sum situation.

20. Our calculations, using $(n-1)/2$ and $(n+1)/2$ are clearly for odd sized groups. Similar results can be shown for even sized groups, but the calculus would have to be developed for $(n/2) + 1$ and $(n/2) - 1$.

21. We will later relax these conditions to consider unequal victims and winners. In other words, non-minimal

We can now derive particular results about when coalitions will reject unjust income distributions using equation **(1)** and substituting in various perspectives on how unjust income is to be evaluated i.e. positing different $\alpha_i f_i(U)$s for the members of the winning coalition.

EGOISM, JUSTICE AND CONDORCET: A SPECIAL CASE ILLUSTRATION

Let us begin with perhaps the simplest case where f_i equals $|u_i|$. In other words, i values a \$1 unjust change in income as having a 'moral' or 'ethical' cost independent of whether it is a gain or a loss. Using this notion, we can ask what level of α could stop political exploitation leading to morally reprehensible redistributions.

When $f_i = |u_i|$ the orientation of the individual towards injustice is purely egoistic: only what happens to *me* counts. So for the individual to reject the transfer, the injustice costs of receiving the additional unjust income must be bigger than the value of that income. But that means that the valuation function must have a value of less than zero for that transfer.

$$V_i(y_i, U) = y_i - \alpha_i |u_i| = \epsilon(n-1)/(n+1)_i - \alpha_i |\epsilon(n-1)/(n+1)I| <0 \quad \textit{Egoism with a concern for injustice } \textbf{(3)}$$

This will happen when $\alpha_i |\epsilon(n-1)/(n+1)| > \epsilon(n-1)/(n+1)$. This reduces to the simple condition of:

$$\alpha > 1 \qquad\qquad\qquad \textit{α to prevent cycles with egoism \&concern for absolute justice } \textbf{(4)}$$

In other words, the cost of the injustice for a gain of ϵ dollars must be greater than the value placed on those dollars.

Clearly that is a rather stringent requirement and a less stringent condition could emerge if one

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coalitions, asymmetries and bystanders will be considered.

notices that concern for injustice often goes beyond mere egoism. One could posit a valuation function $f_i(U)$ that takes into account injustices, not just to oneself, but also to others. Such a broader concern could generate much more leverage against unjust redistribution. In what follows we consider the implications of more 'standard' and normatively plausible conceptions of perspectives on justice and the associated conditions which may preclude unfair redistribution and hence cycles.

IMPARTIAL REASONING, JUSTICE AND CONDORCET

Injustice can enter one's preferences (and hence one's utility function) in myriad ways. For example, rather than evaluating justice from an egoistic perspective, one could take an impartial point of view. Obviously there has been considerable work done on the theory and empirics of justice from an impartial point of view, and needs only to be cited here.[22] Suffice it to say, impartiality has played a significant role in the characterization of justice. Let us analyze the difference that such a shift in perspective can make when reflected in the functional forms of $f_i(U)$, and consider how different perspectives on injustice could affect democratic decisions.

Impartial reasoning changes *whose* interests count in the evaluation of injustice. Individuals taking an egocentric moral point of view as sketched above are concerned only with one's own injustice (either absolute or relative - of which more later). But the broader moral point of view, impartiality, involves taking others' injustice into account directly. That means being concerned with some other's injustice, u_k, perhaps even the injustice suffered by all others. In the latter case, the valuation function, f_i, could take the form $\sum_i |u_i|$. Everyone's unjust income would enter into the evaluation directly. How does this change the calculus?

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22. The reader is perhaps best to start with Harsanyi and Rawls, and then consider the empirical work discussed

The calculation corresponding to equation **(1)** leads us to concentrate on the valuations of member of the winning coalition which become: $V_i(y_i, U_i) = y_i - \alpha_i \sum_i |u_i|$. The losses of each loser are again presumed to be ϵ. Since there are $(n-1)/2$ losers and $(n+1)/2$ winners, each winner again gets $\epsilon(n-1)/(n+1)$ of unjust income, as reflected in equation **(2)**. The impartial winner takes into account both the injustice experienced by each winner and each loser. That sum is simply twice the amount of unjust income taken. The winners, in total, experience an unjust gain of $\epsilon(n-1)/2$ and that is exactly parallel to the total injustice experienced by the losers is $\epsilon(n-1)/2$, So the injustice experienced by an impartial winner, $\sum_i |u_i|$, is simply twice what is lost: or $\epsilon(n-1)$. Since the income gained, $y_i = \epsilon(n-1)/(n+1)$, to forgo an unjust gain, a winner must have a valuation $\alpha_i \sum_i |u_i|$ which is greater in magnitude than the gain. In other words, $\alpha \epsilon(n-1) > \epsilon(n-1)/(n+1)$ Or

$$\alpha > 1/(n+1) \qquad\qquad \textit{α for impartial, absolute justice to prevent cycles} \textbf{ (5)}$$

This is a much less stringent condition than the one implied by an egoistic valuation of justice. It also becomes easier to satisfy as the group size increases. These properties follow directly from the impartial perspective that factors all group members' injustices into the valuation.

RELATIVE V ABSOLUTE JUSTICE

As alluded to above, there are other dimensions along which injustice may be evaluated. The sociologist, Robert Merton (1967) developed a theory of 'relative deprivation' in which individuals measure their welfare relative to the welfare of those in their immediate environs.[23] Relative deprivation can be expanded to develop a notion of 'relative injustice' to be juxtaposed with the

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above. Nagel (1986 and 1991) questions the role of impartial reasoning in fully understanding justice.

23. This notion motivated the studies of Wilkinson, 1997, on the impact of relative deprivation on health status.

notion of absolute injustice sketched above. We then can analyze how that sort of justice concern can block cycles.

Applying a relative injustice evaluation to the egoistic and impartial perspectives already developed, yields a 2x2 typology, as displayed in Table 2.

Table 2: Formalizing Valuation Functions, $f_i(U)$, Regarding Injustice from Different Perspectives		
	Egoistic reasoning	**Impartial reasoning**
Absolute Injustice	$\lvert u_i \rvert$	$\sum_i \lvert u_i \rvert$
Relative Injustice	$[1/(n-1)\sum_k \lvert u_k - u_i \rvert$	$[1/(n-1)] \sum_k \sum_j \lvert u_k - u_j \rvert$

How does a relative evaluation of injustice alter the valuation function, $f_i(U)$ of an individual? If an individual takes an egocentric point of view as sketched above and evaluates injustice in a relative sense she regards the injustices she suffers relative to those suffered by others. Those relative injustices determine the cost term that enters into her utility function. Looking at this algebraically, the individual i is concerned with some term such as $\lvert u_i - u_k \rvert$ for the different individuals k in the group. For her, injustice is a function of those differences. One simple form of this function is the average of the sum of the absolute differences between her and all other individuals, which can be written as $[1/(n-1)]\sum_k \lvert u_i - u_k \rvert$ where i is the individual in question and the k's are the other individuals in the group.[24] That function appears on the bottom row of column 2 of Table 2.

An alternative to an egoistic perspective on relative injustice is an impartial or moral point of

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24. Any relativistic notion of justice evaluation may require the employment of an average to prevent certain peculiar results. Consider for example an unjust taking from i of $1 by a thief, call him j. Then i would have a one dollar injustice relative to some bystander, k. Note that if i only considers the sum of the relative injustice, the larger the group the more numerous the bystanders, the bigger is i's sum of these relative injustices. So some 'normalizing' of the injustice would seem to be called for to properly identify the relative loss. We posit a simple average, 1/(n-1), but other forms are possible.

view. From this perspective *everyone's* injustice counts, and one has to calculate the relative

injustices of everyone, including even all those injustices that *do not involve oneself.* A person taking

such a impartial, relative injustice point of view would be concerned with all such differences.

This is reflected in the double summation ($[1/(n-1)] \sum_i \sum_k |u_k - u_j|$) at the bottom of column 3 of

Table 2.

RELATIVE INJUSTICES, EGOISTIC REASONING AND CONDORCET

Given these new valuation functions we can explore the conditions (weights of the parameters

α in the utility functions) that prevent cyclic majorities. Again, we explore the levels of α required

to prevent majority coalition formation to accept an unjust transfer. Consider first, the case of an

egoistic perspective with a relative justice evaluation: the person is interested in the differences

between the injustice she feels and the injustices experienced by others. To illustrate what is going

on we take the valuation function from Table 2 and use it as the f_i in equation **(1)**, thereby,

generating equation **(6)**.

$$V_i(y_i, \boldsymbol{U}) = y_i - \alpha_i/(n-1) \sum_k |u_k - u_i| \qquad \textit{Moral egoism \& relative justice (6)}$$

How then, is this notion of relative
injustice to be interpreted? Consider a
status quo where all income, and hence all
income differentials, are just and then
redistribute it unjustly. How do the

Table 3: Illustrative Distributions with Transfers			
Individual's Income (Income, unjust 'shortfall')			"Story"
i	*h*	*k*	
2, 0	2, 0	2, 0	original distribution
2.5 , .5	2.5 , .5	1, -1	post unjust transfer dist.

transfers in income fit into this picture? Specifically, how does an individual evaluate these

transfers and changes in income? To help understand how injustices enter into the evaluations,

consider the illustration in Table 3 in which the first entry in each cell stands for actual income

and the second unjust income: (y_i, u_i)

The original situation is one in which each individual starts with no injustice: their income y = $2 is all just and hence their unjust incomes, u = 0. Each individual, i, evaluates the situation as $V_i(2) = 2$.

Now suppose that a gang of 2 (say, i and h) takes $1 from k and divides the booty equally between themselves. There are now differences in individual incomes, and these differences are injustices. The differential terms at the end of expression **(6)** above are no longer zero. After all, k would now have $1 *less* than her *just* income (hence $y_k = \$1$, and $u_k, = -1$). k now compares this shortfall to that of each of the other 2 individuals. Each of the gang members will have u = +.5 versus k's u= -1, giving a differential of 1.5 between k and each of the exploiters. The absolute value of the average differential of unjust incomes between k and the two others persons $(|u_k - u_{i,h}|) = 1.5$. And from the perspective of the thieves, how do things add up? Specifically since they divided the booty equally, the relative injustice for each thief is zero in relation to her thieving partner but it is $|u_{i,h} - u_k| = 1.5$ with k. Hence, the *average* differential each thief experiences is half that, or .75. When would i or h be unwilling to engage in this sort of self enriching unjust behavior? That would occur when the gain in income (.5) was not worth the loss due to the cost of the (weighted) unjust income differential $(-\alpha)(.75)$. As long as for i, or for h, $\alpha_{i,h} > 2/3$, the thief wouldn't find the payoff worth it.

Table 4: Values of α to Avoid Cycles		
Type of Justice	**Egoistic perspective**	**Impartial perspective**
Absolute	$\alpha > 1$	$\alpha > 1/(n+1)$
Relative	$\alpha > (n-1)/n$	$\alpha > (n-1)/n(n+1)$

In the next two sections we derive the specific conditions for α to prevent majority cycles with relative egoistic (and then impartial) injustice concerns. The conditions on α for the four different perspectives on injustice are summarized in Table 4.

Consider equation **(6)**. The question to be answered remains, "What are the conditions that make it too costly for those receiving the unjust transfers to accept the transfers?" Each winner's gain, from equation **(2)** is always $\epsilon(n-1)/(n+1)$, and it must be less than the decrement to her welfare from the injustice. With an egoistic relative perspective on unjust the valuation of the income injustice becomes $1/(n-1)\sum_k |u_k - u_i|$. For a winner to reject a transfer, the monetary gain must be less than this cost:

$$\epsilon(n-1)/(n+1) < \alpha_i/ (n-1) \sum_k |u_k - u_i| \quad \textit{Moral Egoistic Relativistic Winner Prefers No Unjust Transfer} \textbf{(7)}$$

Given that all members of the winning coalition get the same payoff, those differentials among the winners equal zero. But the differentials between i and any member who has had monies taken will be $\epsilon + \epsilon(n-1)/(n+1)$ which simplifies to $2n\epsilon/(n+1)$. There are $(n-1)/2$ victims of injustice and hence $(n-1)/2$ such differentials. So we have $\epsilon(n-1)/(n+1) < [\alpha_i /(n-1)][2n\epsilon /(n+1)]$ $[(n-1)/2]$ which simplifies to

$$\alpha > (n-1) /n \qquad\qquad \textit{α for Egoistic Relativism to prevent Cycles} \textbf{(8)}$$

This equation allows us to understand that the relatively low value for α (i.e $\alpha > 2/3$) in our previous example came from the size of the group. In a group of 3, α need only be $> 2/3$ for the support for the initial income distribution to be stabilizing. In general, as the group size increases, the α necessary to produce a stable outcome will be increasing with a limiting value of 1.

RELATIVE INJUSTICES, AN IMPARTIAL PERSPECTIVE AND A CONDORCET WINNER

Consider, finally, the case of an individual taking an impartial perspective on relative injustice.

Again, each winner gets $\epsilon(n-1)/(n+1)$ (i.e. y_i) and as above, differentials between winners and losers simplify to $2n\epsilon/(n+1)$. What is different is *how many* differentials are to be taken into account. There are $(n+1)/2$ winners, each with a non-zero difference with $(n-1)/2$ losers. The product of these constitute all the differences of the winners with the losers. But similarly, there are the same differences from the losers perspectives. So the final number of differences which must be considered are $2(n+1)(n-1)/4$. So for each winner, equation **(1)** becomes:

$V_i(y_i, \boldsymbol{U}_i) = \epsilon(n-1)/(n+1) - [\alpha_i/(n-1)](2(n+1)(n-1)/4)(2n\epsilon/(n+1)]$ and the condition for a winner to reject an unjust transfer simplifies to:

$$\alpha > (n-1)/n(n+1) \qquad\qquad \textit{α for impartial reasoning relative injustices to prevent cycles } \textbf{(9)}$$

From **(9)** it follows that as the group size (n) gets large α can approach 0. This is the least demanding condition on alpha because the valuation function for injustice takes into account *all* relative injustices in the society as a result of the impartial perspective.

SOME EXTENSIONS

We have demonstrated that a shared concern for injustice can preclude cycling in redistributional issues in a majority rule decision system. The stringency of the required level of concern is a function of the perspective taken on justice. When an individual is egocentric and is only concerned with the injustices *she* is subject to, (see Table 4, 5) the incentives to forgo unjust transfers are minimal. The transition to an impartial point of view (the right hand column) changes the incentives dramatically: the individual has a much greater incentive to refrain from any unjust actions. Finally, an evaluation of justice that is relative rather than absolute also increases the weighting on justice somewhat in both columns.

But it is surely not lost on the reader that the analyses we have presented are polar cases and

nested in a majority rule decision context. The assumptions fix the size of the winning and losing

coalitions and divide society is into two mutually exclusive categories: winners (exploiters) and

losers.[25] It is possible to relax these assumptions to provide an analysis of a more general context.

We can allow the size of the winning and losing coalitions to vary and can also include a category

of bystanders to the redistribution process.

Just as we assume that there is consensus on a justice vector *J* without specifying the *content* of

the underlying notion of justice, we can assume some decision rule for political decisions without

specifying its content. All we need do is posit a set of n individuals, and a decision rule which

generates three exclusive groups, winners, (W), losers (L) and bystanders (B).[26] One possible

interpretation of this threefold division of the society in question is that some members simply do

not participate actively in the redistribution, either as losers, or recipients of transfers. They would

only be affected by externalities of redistribution, and would possibly generate secondary

externalities by their status relative to the other active members. To illustrate how this might

work, we have reworked our analysis of the four perspectives presented above in terms of

winning coalitions of size W, losing coalitions of size L and bystanders representing the residual

individuals (if any) numbering (n - W - L). Again, we can derive the conditions of α which

preclude redistributional cycles. The derivation appears in Appendix. The conditions are

represented in Table 5.

There are a few implications of this, more general, solution to the level of α which precludes

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25. Similarly, we have assumed that if one relativizes one's experienced injustices, one does so with reference to

everyone in the group. Of course, this need not be the case.

26. The inclusion of Bystanders does not limit the generality of the results derived. By setting their number to zero

one can see what values of α preclude cycles without bystanders.

unjust redistributions. First, the sizes of the winning and losing coalitions make no difference in

the size of α if individuals take an egoistic perspective. This follows directly from the fact that

such individuals care only about what is happening to themselves, either absolutely or relatively.

The sheer value of the transfers to themselves and others is all that matters. So column 1 of Table

5 is the same as column 1 of Table 4.

By contrast, individuals who take an impartial point of view are affected by the distributional

characteristics of the transfers. This results in αs which are a function of the size of the winning

coalition for individuals who take an impartial absolute perspective on injustice.

For such individuals, the larger the size of the winning coalition, the smaller the size of α necessary to preclude an unjust transfer. This is implied by the introduction of more members into the

Table 5: Values of α to Avoid Cycles		
Type of Justice	**Egoistic perspective**	**Impartial perspective**
Absolute	$\alpha > 1$	$\alpha > 1/(2W)$
Relative	$\alpha > (n-1)/n$	$\alpha > \dfrac{(n-1)}{2Wn + 2W(n-W-L)}$

winning coalition which dilutes the share of the constant level of transfer going to each winner.

At the same time an impartial perspective makes each winner enter all the injustice in the society

as a cost in his or her utility function, so, inasmuch as there are fewer losers, it takes a lower α to

compensate a winner for any given ϵ taken from each of the set of losers.

By generalizing beyond simple majoritarian democracy we can see why and when democracy

generates pressure for justice. Assuming that there is a moral conception of the task, the relative

size of the winning coalition affects the ease of generating a just outcome *for any distribution of α's and any impartial point of view.* As the size of the winning coalition, W, goes up, the rewards for unjust political plunder decrease to the individual actor in W. Thus, the coalitional demands of democracy tend to push it toward the achievement of justice. Note that we would expect that those democracies with institutions that don't generate large, unified winning coalitions will have a harder time generating just policy outcomes.[27]

Finally, if an individual takes an impartial perspective on a relative evaluation of injustice, the sizes of both the winning and losing groups affect the size of the α necessary to preclude redistribution. Here, since relative injustice is the measure, the distributional properties of gains and losses is important in determining the cost term in the utility function. Again, the more winners there are, the smaller the α necessary to preclude redistribution for the same reasons noted in the preceding example. However, in this case, the number of bystanders enters in, with the requisite α necessary to preclude redistribution falling as the number of bystanders increases. As before, the α required falls with increasing group size.

The general message conveyed by the table is that when group members take an impartial point of view, the sizes of the coalition components matters.[28] Perhaps most importantly, when the decision mechanism allows for smaller numbers of winners in the winning coalition, the αs needed to get the winners to abjure unjust transfers increase in size. Thus, the moral point of view taken and the political decision rules governing who wins and who loses determine the

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27. Mancur Olson (1990, 1993) concluded similarly that the achievement of public welfare would vary positively with the degree to which the governing coalitions are encompassing.

28. In all of our examples the moral point of view treated is an impartial point of view. There are other moral points of view that could be considered.

constraints on α to be effective. One important observation is that decision rules that require broader based support for decisions to be effective, are, ceteris paribus, less likely to result in unjust transfers.

DISCUSSION

By assuming justice to be universally understood and endogenous to individuals we have shown when weightings of justice lead to reasonable and interpretable outcomes in a democratic process. This allows us to go beyond questions of democracy, cycles and justice. And it is to this more general level that we now turn.

Our approach is in the individualistic tradition of welfare economics but differs in that we posit a consensus on a specific aspect of other-regarding evaluations. Arrow himself was looking for a social welfare function based on the 'wide' or 'total' preferences of the individual members of the group. He never restricted this set of preferences to egoistic, or self-interested, preferences. The conditions we have identified as sufficient to preclude cycling on redistributive issues constitute a new direction along the path of restricted preferences that Arrow and others have noted as a possible way of developing a credible Social Welfare Function. Indeed, it appears to us, that to develop a normatively interesting SWF it may be necessary to consider, explicitly, the normative concerns *in the preferences of the individuals* rather than looking *only* at the *formal properties of preferences* or the *normative properties of the aggregation system*. The need for some commonality, or agreement, regarding those moral concerns, is not dissimilar to the common understanding of the political space underlying single peaked preference results.[29] Arrow, 1977, makes a related

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29. The relationship between social welfare and social choice motivated both Arrow and us. The issue goes far beyond avoiding cycles to searching for a relationship between individual choices, collective outcomes, and social welfare that can be established by institutional design.

suggestion when he shows that *if* we assume that there is a consensus on whether one individual in some situation is better or worse off than another in a different situation, then the only SWF that works is a lexical minimax or maximax one. Perhaps the lesson from Arrow is that 'from nothing one gets nothing.' *Something* normative must be added to the preferences of the individuals to get further. We are proposing how one can modify the pessimistic results of the social choice literature by adding normatively interesting material into preferences in order to get a better handle on the issues of social welfare.

RETHINKING JUSTICE:

If we have put justice front and center in the development of social choice, it is deliberate. Social choice became a field of interest, not because of cycles, but because of the incoherent relation that Arrow established between social choice and social welfare. The real 'cost' of the Arrow finding is the non-interpretability of democratic social choice in social welfare terms (Mackie, 2003; Riker, 1982).[30] With cycles, one is confronting incoherent choice. Cycles sever the normative link between the individuals' preferences and the social choice. It makes it difficult to evaluate the performance of political decisions in terms of satisfying citizens' preferences. We attempt to tackle that head on by arguing first: social welfare can be understood, at least in large part, in terms of justice. To see this, one must go into avenues that are tangential to the arguments of this paper.

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30. In electoral models there can also be a direct interpretation of social welfare from the individuals' preferences, as long as voting is probabilistic (Coughlin, 1988). In those cases, with candidates attempting to maximize expected vote maximization, the result can be interpreted in terms of utilitarianism. Given expected vote maximization, individual's preferences are weighed together as a function of their probabilistic 'responsiveness' to distances from their ideal points. In our analysis, which is not probabilistic, responsiveness is captured by $\alpha_i f_i$.

Theoretically, Rawls (1971), and subsequently many others, tried to justify the lexicographic concerns for the poor, or the worst off, in the development of theories of distributional justice. These arguments are clarified when they are considered from the vantage point of experimental data: *universally* individuals who consider the problem from an 'impartial' point of view, choose to have a social contract that establishes a floor, and then to consider other elements such as just deserts. We might wonder, what is it about the floor that makes it a universal concern?

Within the traditional Arrovian framework there are few evaluative measures of political systems beyond 'Pareto optimality' because of a reluctance to introduce interpersonal comparability of individual welfare. Of course there are other 'rough indicators' relevant to specific policy goals such as the number of violent deaths, per capita income, life expectancy, and penal detentions. But their evaluative status doesn't link back to the preferences of the individual citizens: they aren't endogenous via preferences alone (Schotter, 1984). The nice thing about an approach which generates a concern for justice based on consensus is that it focuses on something substantive: what justice *is*. And such a focus leads to a concern for a floor, and leads us to wonder why the floor is the focus of attention.

Asking the question in this manner brings one directly to the concept of needs: basic needs.[31] Satisfying basic needs (such as warmth, health, hunger, etc.) become a prior concern to other things, and as such, it is understandable why it is universally identified as an essential aspect of

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31. On this, the recent work of Gillian Brock is most central. She argues that only basic needs justify a lexicographic concern with a floor, and that, as an extension, basic needs must be considered a motivator for rights. In any case, she builds on the excellent work of Braybrooke (1987) and Doyle and Gough (1991). Braybrooke points out that Pareto Optimality isn't what is left with utilitarianism without interpersonal comparison, except if we agree that preferences rather than needs are to replace utility (page 175). He also conjectures that a concept of needs instead of preferences could get one around some of the paradoxes of social choice theory (pp. 27, 184-6).

justice. A rough approximation of social welfare then *would* put needs lexicographically ahead of other elements of welfare.[32] Indeed, if all peoples hold this dear, one must ask what prevents such concerns from being effectively expressed politically, and fulfilled with high priority, in democratic political systems. Focusing on needs, as *the* lexicographically prior element of justice allows us to reevaluate both the role and the priority of justice.

There is yet another possible benefit of democracy as a promoter of justice. When there exists a shared sense of justice, increasing injustice in a society is a public bad.[33] It affects everyone, albeit differently. The costs of redressing any particular incremental move towards more justice may be too great for any one individual to be willing to undertake. Yet the individual might well be willing to vote to have certain injustices prevented or righted.[34] Thus justice might well be expected to become an openly debated public good, in part, secured by the state.

Of course, there is more to our understanding of justice than is contained in some set of preference functions and an agreement on the content of *J*. Few of us would be happy with a universal conception of these functions that had dummy variables for race, slaves, and gender in it. In other words, there may be other variables that ought to serve as side constraints to the notions of justice (Nozick, 1974).

THE ROLE AND PRIORITY OF JUSTICE

In some sense, the results of democratic systems can *only* be tied to notions of welfare when

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32. We say rough approximation because incentives matter in the achievement of social welfare and justice. That being said, so do tradeoffs between such needs, incentives, just deserts, and efficiency

33. Thurow, 1971 suggested that individual attitudes toward income distributions could lead one to consider the income distribution as a public good

34. This is also the perspective taken in Wittman (2002).

there is consensus as to what *constitutes* such welfare. A shared sense of justice may be a necessary component of any such consensus.[35] Here deliberative democracy can play a role. It would appear, from the work of Miller (1999) and others (see above), that public deliberation among the citizenry and the politicians can help uncover the agreement regarding *J* and can help the evocation of a moral point of view by a sizeable proportion of the population.[36]

Rather than considering justice as a simple element to be weighed in social welfare, justice as understood in keeping with the universal focus on the floor, and hence needs, can be thought of as *the* metric for the evaluation of democratic institutions, and social outcomes. This might not be obvious, until it is recalled that although none of the developed democracies have in fact delivered a serious welfare floor to all their citizens, there is a great deal of variance in their performance.

Of course, cultural differences have long been argued to be the reason for the differences in delivery of justice. Some peoples emphasize just deserts more than others. What such an explanation leaves out, of course, is the underlying consensus that seems to exist among members of our species as to what constitutes justice. As such, it then becomes interesting to ask what impeded the expression, or evocation, of those preferences for justice?

FRAMING

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35. Indeed, this seems to be the motivation of David Miller's (1999) conjecture that pluralist democratic societies are less likely to deliver social justice than non-pluralist ones.

36. Using data from a set of experiments we have reported elsewhere (Frohlich and Oppenheimer, 2003) we have tested this conjecture. Our analysis shows that under all of our experimental conditions (both computerized and face to face discussions; and in prisoner dilemmas or pure cooperative games) self reported intentions and self-described behavior are very significantly (t-tests p always < .0001) more motivated by moral sentiments when there is discussion.

Since we have traveled a fairly long analytic path, it may be useful to review the ingredients of the argument: a set of valuation functions f_i's , individual valuations of injustice, α_i's, a shared sense of both justice (***J***) and knowledge of the status quo (***Y***); a set of political institutions or rules of the game that determine the sizes of the winning, bystander, and losing coalitions (W, B, and L). We might say that a political system can move the system toward the ideal state of justice when the values placed on it are sufficiently high given the rules of the game (i.e. the determinants of W, L).

The distribution of α's is not the only factor determining the impact of a sense of injustice on the decision. The outcome is also a function of the points of view adopted by the set of individuals. Individuals who adopt moral points of view will have different valuation functions and hence radically lower thresholds of α to make the justice concern decisive. And adopting a moral point of view can be influenced by how an issue is framed. *Ex ante,* there is no reason to assume that any one specific perspective on justice will predominate and determine the action chosen. Any given situation can be perceived from more than one perspective and cues available in the presentation of the situation can affect the decisions (Frohlich and Oppenheimer, 2000, 2001 and, see Kahneman and Tversky, 1979; Tversky and Kahneman, 1981). Specifically, normative elements may be evoked - or repressed - by framing effects. If a situation is perceived as involving a moral question, the individual's revealed preference is more likely to include elements giving weight to considerations such as justice. For example, Roth (1995) discusses evidence that market decisions decrease the extent to which normative components enter into individuals' decisions. Other experimental results, cited above, have shown that framing effects can affect the normative components that enter decisions. So, whether an individual adopts an egoistic or impartial point of view is likely to be sensitive to the way in which the decision is

framed. Whether the perspective is absolute or relative may also be sensitive to framing effects.

It may be reasonable to evaluate democratic institutions on the basis of their propensities to frame issues in such a way that concern for others, and hence justice, are evoked. As noted above, political actors can frame issues as having moral content or not, but it may be that certain institutional characteristics facilitate, or inhibit such behavior. For example, take health care delivery: an issue over which people have a complex set of preferences that have both egoistic and other-regarding elements. Then a set of institutions (or discourse) that evokes only egoistic elements as opposed to institutions that also evoke some normative elements is likely to generate inferior social welfare results. Thus, discourses that frame the issue in a more normative context could be argued to be more desirable than the former. One implication of our analysis is that the suppression of impartial perspectives by framing a political issue selfishly can not only generate inferior results, it may, in distributive matters, encourage cycles.

So, in any real world situation, the tolerance for injustice will be largely determined by the framing of the issue as either one to be thought about in an egoistic (self-interested) fashion or in a moral fashion. Since it is the political actors, including the media, who frame the issues they can significantly affect the outcomes of the political process by their initial framing of the decision.[37] This raises the question of what aspects of political questions allow some issues to be more easily framed within a moral context and thus be more amenable to stable resolution.

In conclusion, it appears that a level of consensus, and a conception of evaluation of policy from an impartial point of view, can lead one to specifiable conditions for avoiding the difficulties

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37. Of course how people perceive a problem depends on how it is presented. The media's depiction of the poor refugees from Hurricane Katrina (2005)'s hit on New Orleans has apparently led to many Americans adopting a moral point of view regarding the delivery of disaster relief.

of incoherent social welfare indicators and cyclic choice. A concern for justice can, when mediated by accommodating political institutions, help fulfill these two conditions nicely. A closer look at justice should give those of us interested in institutional design and policy evaluation a leg up in how to proceed.

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APPENDIX SHOWING CONDITIONS OF α TO PREVENT CYCLES WITH DIFFERING

CONCEPTIONS OF INJUSTICE

Since we are interested in the *political* stability of the income distribution, we considered the possibility of any decision rule which allows a winning coalition to take ϵ from a member of a losing group, while allowing for a third group of bystanders to neither gain nor lose income from a transfer.[38] The argument developed above about threshold values for α to end cycles can be generalized to these general conditions. Allow the winning coalition to have size W, and allow the group of individuals who will be slated to suffer the injustices to have size L. If $W + L < n$ (the size of the group), this will leave bystanders, B (of size $n - L - W$). For example, 1 person in a 5 person group might appropriate \$100 unjustly from one victim, leaving 3 bystanders.

We continue, throughout, presuming the L losers share equally in the loss: they each lose ϵ. Similarly the W winners each get an equal share of what has been redistributed or ϵL each winner gets $\epsilon L/W$. Or

$$y_i = \epsilon L/W \qquad\qquad \textit{Winner's unjust gains from unjust transfer \textbf{(A1)}}$$

Now we can generalize the story of when winning coalitions will reject unjust income distributions using equation **(2)** by substituting into the equation any one of the perspectives on unjust income recalling that was is required is that the winner's gain is less than the decrement to her welfare from the injustice. So consider the 4 perspectives regarding the evaluation of J to derive the associated values of α.

CONDITIONS FOR AN ALPHA FOR REFUSAL WITH EGOISTIC JUSTICE:

The case of the absolute valuation by an egoistic evaluator of justice is straightforward. After all, the argument from above remains unchanged. Here i's gains, from an unjust transfer of y_i lead

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[38] Some groups, for some reasons, (perhaps religious imperatives or tradition or coalitional needs) may not be party to the redistribution directly, but their relative status may impact others' decisions.

to a moral cost associated with an unjust income of equal magnitude. Hence, for the weight of the moral cost to be sufficient to prevent the acceptance of the income, α needs to be >1.

Consider now the case of egoistic relative justice. Recall, from equation **(6)** that in such a case:

$$V_i(y_i, \boldsymbol{U}_i) = y_i - [\alpha_i/(n-1)] \sum_k |u_k - u_i|$$

What is required for a member of a winning coalition with an unjust gain, y_i, to value it less than the decrement to her welfare from the injustice, $\boldsymbol{U}_i$? To compute this, we need to examine the relative differences between i and each of the three classes of group members: those that are in the group of winners W; those that are in the group of losers L, and those that are members of the bystanders, B. Clearly, there are no relative differences of injustice between members of the winning coalition, so we need to consider only the injustice differences between 1) the typical winning and losing coalition members, and 2) the typical winning and bystander coalition members. The victim, $l \in L$, loses ϵ, and the winner, $i \in W$, gains $\epsilon L/W$. Hence the difference between a single pair of these winners and losers is merely their sum or: $\epsilon(L+W)/W$. But there are L losers, and in averaging the relative differences, i must take this into account. So the difference between i and all the losers is:

$u_{i \in W, \forall l} = \epsilon\, L(W+L)/W$ *Total difference between the winner and all losers* **(A2)**

Consider now the difference between the winner, i, and the bystander, b. Here the differential is merely the gain $y_i = \epsilon L/W$. Then, given that there are n - L - W members of B, the difference between i and all the bystanders is:

$u_{i \in W, \forall b} = \epsilon L(n - L - W)/W$ *Total difference between the winner and all bystanders* **(A3)**

Adding **(A2)** and **(A3)** gives us $\sum_k |u_k - u_i|$, which simplifies to $\epsilon Ln/W$ and then we can identify the threshold for α as:

$$\alpha > (n-1)/n \qquad\qquad \textit{Refusal value of Alpha for egoistic relativist} \textbf{ (A4)}$$

CONDITIONS FOR AN ALPHA FOR REFUSAL WITH IMPARTIAL REASONING:

Recall that with impartial reasoning over absolute levels of injustice, $V_i(y_i, \boldsymbol{U}_i) = y_i - \alpha_i \sum_i |u_i|$. So here we need consider the value of u for the 3 classes of group members. For $b \in B$ u is 0. For $w \in W$ the value is the same as his gain or $\epsilon L/W$. Since there are W such individuals the total u to that subgroup is merely ϵL. And to l, members of L, the losers, each of whom lose ϵ, the total loss is merely, also ϵL. So: $\alpha \sum u_i > y_i$ or $\alpha 2\epsilon L > \epsilon L/W$. This simplifies to

$$\alpha > 1/2W \qquad\qquad \textit{Refusal value of Alpha for impartial reasoning absolutist} \textbf{ (A5)}$$

This then leaves one other group to be considered: the impartial reasoning relativist. For her, $V_i(y_i, \boldsymbol{U}_i) = y_i - [\alpha_i/(n-1)] \sum_k \sum_i |u_k - u_i|$.[39] This can be shown to be similar to the calculation for the egoistic relativist, and many elements stay the

Table A1: Total valuation of Injustice elements for an impartial reasoning relativist		
Injustice differences	Total number of differences	Product of difference and number
$U_{w,l} = \epsilon(W+L)/W$	2WL	$2\epsilon L(W+L)$
$U_{w,b} = \epsilon L/W$	2W(n-L-W)	$2\epsilon L(n-L-W)$
$U_{b,l} = \epsilon$	2L(n-L-W)	$2\epsilon L(n-L-W)$
full sum of u considered by i		$2\epsilon L(2n-L-W)$

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39. The similarity between this and the formula for the Gini Index ought to be noted, given their similar motivation (see the discussion in Sen, 1973).

same. Basically, recall, $i \in W$ must consider the injustice felt by each of the subgroup members, and weight them by the number of subgroup members. Hence, the difference between each group is 'double counted' as it is seen by the member of both sides of the relationship. So we add the values of *all* the differences between, for example, *w*, a winner, and *b*, a bystander. We then note that our impartial reasoning individual must consider how many such bystanders there are for each winner or he will multiply the difference by W and (n - W - L).

The resulting calculation is fleshed out in Table **A1**. Solving the expression for α gives us:

$$\alpha > (n-1)/2W(2n - W - L) \qquad \textit{Refusal value of Alpha for impartial reasoning relativist} \textbf{ (A6)}$$

DISCUSSION

The above generalizations of the results can be easily checked to be the general form of the earlier results. One need only recalculate the arguments for the special case of $L = (n-1)/2$ and $W = (n+1)/2$. What we have shown is that only impartial reasoning is really affected by changing the sizes of the subgroups of winners and losers in the society. Egoists are motivated by their own winnings, and the costs (including moral) of these gains.